MATH 556 - ASSIGNMENT 3

To be handed in not later than 5pm, 11th November 2008.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Suppose that two random variables X and Y (defined on the same probability space $(\Omega, \mathscr{E}, P)$ are such that

$$F_X(t) \leq F_Y(t) \qquad \text{for all } t \in R$$

or equivalently, if $B = \{\omega \in \Omega : X(\omega) \geq Y(\omega)\}$, then $P(B) = 1$. Then X is termed *stochastically greater* than Y, and we say that X *stochastically dominates* Y.

(a) Prove that X is stochastically greater than Y **if and only if**

$$\mathbb{E}_{f_X}[g(X)] \geq \mathbb{E}_{f_Y}[g(Y)]$$

for any non-decreasing real function g for which the expectations exist.

Hint: for the "if", consider $g_c(x) = \mathbb{I}_{[c,\infty)}(x)$ for arbitrary real c. For the "only if", use the second part of the definition of stochastic dominance.

6 MARKS

(b) Show that, if $X \sim Poisson(\lambda)$ and $Y \sim Poisson(\mu)$, and $\lambda \geq \mu$, then X is stochastically greater than Y.

4 MARKS

2 One method of comparing two pdfs f_1 and f_2 is to compute the following distance measure

$$\mathbb{H}(f_1, f_2) = \frac{1}{2} \int_{-\infty}^{\infty} \left(\sqrt{f_1(x)} - \sqrt{f_2(x)} \right)^2 \, dx$$

where $\sqrt{.}$ indicates **positive** square root.

(a) Show that $0 \leq \mathbb{H}(f_1, f_2) \leq 1$ for arbitrary f_1, f_2.

2 MARKS

(b) Show that for arbitrary f_1, f_2

$$\mathbb{H}(f_1, f_2) \leq \min\{\mathbb{K}(f_1, f_2), \mathbb{K}(f_2, f_1)\}$$

where $\mathbb{K}(f_1, f_2)$ is the Kullback-Leibler divergence between f_1 and f_2.

4 MARKS

3 Consider the discrete pmf, f_X, defined for $i = 1, 2, \ldots$ by

$$f_X(x_i) = p_i$$

where $\mathbb{X} = \{x_1, x_2, \ldots\}$, with $x_i > 0$ for all i. Suppose that

$$\mu = \mathbb{E}_{f_X}[X] = \sum_{i=1}^{\infty} p_i x_i < \infty.$$

Show that

$$\mu e^{\mu} \leq \sum_{i=1}^{\infty} p_i x_i e^{x_i}.$$

3 MARKS