MATH 556 - ASSIGNMENT 2

To be handed in not later than 5pm, 7th October 2008.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Suppose that $X_1 \sim Geometric(\theta_1)$ and $X_2 \sim Geometric(\theta_2)$ are independent random variables. Find the pmf of random variable Y where $Y = X_1 + X_2$.

3 MARKS

2 Suppose that $U \sim Uniform(0,1)$ is a continuous random variable. Find a transformation g such that $X = g(U)$ is a continuous random variable that has the pdf

$$f_X(x) = \frac{2}{\pi}\frac{1}{\sqrt{1-x^2}} \qquad 0 < x < 1$$

and zero otherwise. Sketch this pdf.

3 MARKS

3 Suppose that X_1 and X_2 are random variables with joint pdf given by

$$f_{X_1,X_2}(x_1,x_2) = c|x_1|\exp\left\{-|x_1| - \frac{x_1^2 x_2^2}{2}\right\} \qquad (x_1,x_2) \in \mathbb{R}^2$$

Find the marginal pdf of X_1, and the conditional pdf of X_2 given $X_1 = x_1$, for appropriate values of x_1. *Take care to define the pdfs for all real values of their arguments. Compute the value of constant c*

4 MARKS

4 (a) Consider random variable X with probability function P_X and cdf F_X. The indicator random variable for set B, $\mathbb{I}_B(.)$, is a transformation of X, and is defined by

$$\mathbb{I}_B(X) = \begin{cases} 1 & X \in B \\ 0 & X \notin B \end{cases}$$

Find the pmf/pdf of $\mathbb{I}_B(X)$, and the expectation of $\mathbb{I}_B(X)$.

2 MARKS

(b) The expectation of any random variable with pmf/pdf f_X can be approximated to arbitrary accuracy (under mild conditions) by a *Monte Carlo* simulation procedure: a large sample of simulated values $x_1, \ldots, x_N$ are generated from f_X, and then the expectation is approximated by the sample mean to produce the approximation $\widehat{\mathbb{E}}_{f_X}[X]$.

$$\widehat{\mathbb{E}}_{f_X}[X] = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Using a suitable computer package (R or MATLAB say), use the result in (a), and a Monte Carlo procedure, to approximate the probability $P_{\underset{\sim}{X}}[\underset{\sim}{X} \in B]$ if $\underset{\sim}{X} \sim \mathcal{N}_3(0,\Sigma)$ with

$$\Sigma = \begin{bmatrix} 1.0 & 0.2 & -0.5 \\ 0.2 & 2.0 & -0.1 \\ -0.5 & -0.1 & 2.0 \end{bmatrix}$$

and B is the set $\{\underset{\sim}{x} \in \mathbb{R}^3 : x_1^2 + x_2^2 \leq x_1 + x_3\}$. Give your code, and tabulate the results of five replicate Monte Carlo runs with $N = 10000$.

8 MARKS

Note: In `R`, the function `mvrnorm` from the library `MASS` can be used to simulate from the multivariate normal distribution. Other basic functions in `R` that may be useful are

`matrix`, `c`, `sum`, `mean`

In `R`, the `help()` function can be used to see how each function is used.

A basic introduction to `R` can be found (for example) at

http://cran.r-project.org/doc/manuals/R-intro.pdf