MATH 556 - ASSIGNMENT 1

To be handed in not later than 5pm, 18th September 2008.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Consider the probability space $(\mathbb{R}, \mathscr{B}, \delta_a)$, for real constant a, where, for $B \in \mathscr{B}$

$$\delta_a(B) = \begin{cases} 1 & a \in B \\ 0 & a \notin B \end{cases}$$

The probability measure $\delta_a(.)$ is referred to as the *point mass measure* at a.

Show how the probability distribution of **any** discrete random variable can be constructed from point mass measures.

4 MARKS

2 Suppose that F_1 and F_2 are two continuous cdfs with corresponding pdfs f_1, f_2, where, for some real c, $F_1(c) < F_2(c)$. Consider the random variable X with cdf

$$F_X(x) = \begin{cases} F_1(x) & -\infty < x < c \\ F_2(x) & c \leq x < \infty \end{cases}$$

Show that F_X is a valid cdf, and find the corresponding probability measure P_X. Use the notation P_1 and P_2 for the probability measures associated with F_1 and F_2, that is, for $i = 1, 2$ and $x \in \mathbb{R}$, $P_i((-\infty, x]) = F_i(x)$.

4 MARKS

3 The *hazard probability function*, h_X, for a discrete random variable X with support $\mathbb{X} = \{1, 2, 3, \ldots\}$ is defined by the formula

$$h_X(x) = \frac{f_X(x)}{1 - F_X(x-1)} \qquad x \in \mathbb{X}$$

where f_X and F_X are the pmf and cdf for X. Find the hazard probability function for the *Geometric* distribution, where

$$f_X(x) = (1-\theta)^{x-1}\theta \qquad x \in \mathbb{X}$$

and zero otherwise, where $0 < \theta < 1$.

4 MARKS

4 Establish whether the following functions define valid distribution functions. If they do, also find the corresponding joint mass or density functions.

(a)

$$F(x_1, x_2) = \begin{cases} 0 & x_1, x_2 < 0 \text{ or } x_1 > x_2 \\ 1 - e^{-(x_1 + 2x_2)} & 0 \leq x_1 \leq x_2 < \infty \end{cases}$$

4 MARKS

(b)

$$F(x_1, x_2) = \begin{cases} 0 & x_1, x_2 < 0 \\ 1 - e^{-x_1} - x_1 e^{-x_2} & 0 \leq x_1 \leq x_2 < \infty \\ 1 - e^{-x_2} - x_2 e^{-x_2} & 0 \leq x_2 \leq x_1 < \infty \end{cases}$$

4 MARKS