

MATH 545 - PRACTICE MID-TERM

All questions carry 10 marks.

The total mark available is 40; rescaling of the final mark may be carried out.

In the following questions, $\{Z_t\} \sim \text{WN}(0, \sigma_Z^2)$, with $t \in \mathbb{Z}$; ACVF refers to the *autocovariance* function, ACF refers to the *autocorrelation* function.

1. Suppose that time series process $\{X_t\}$ has a linear process representation

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j Z_{t-j} \quad t \in \mathbb{Z}$$

for constants $\{\psi_j, j = 0, \pm 1, \pm 2, \dots\}$ such that $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$.

Derive the form of the ACVF for $\{X_t\}$, $\gamma_X(\cdot)$, in terms of $\{\psi_j, j = 0, \pm 1, \pm 2, \dots\}$ and σ_Z^2 , and use it to derive the ACF of the process defined by

$$X_t = Z_{t+1} + Z_t + Z_{t-1}.$$

2. State, giving brief reasoning, whether the following time series processes are stationary and causal:

(a) $\{X_t\}$ defined implicitly for all t by the equation

$$X_t = 1.2X_{t-1} - 0.35X_{t-2} + Z_t.$$

(b) $\{X_t\}$ defined implicitly for all t by the equation

$$X_t = 3X_{t-1} - 3X_{t-2} + X_{t-3} + Z_t.$$

3. Find the form of the ACF, $\rho_X(\cdot)$, for the stationary time series process $\{X_t\}$ defined by

$$X_t = \frac{1}{2}X_{t-1} + Z_t + \frac{1}{2}Z_{t-1}$$

and show that the process $\{X_t\}$ defined by

$$X_t = \frac{1}{2}X_{t-1} + Z_t - \frac{1}{2}Z_{t-1}$$

is a zero mean white noise process.

4. Consider the time series process $\{X_t\}$ defined for all $t \in \mathbb{Z}$ by

$$X_t = U \cos(\pi t/4) + V \sin(\pi t/4)$$

where U and V are uncorrelated random variables with expectation zero and variance one.

Show that $\{X_t\}$ is stationary, and find its ACVF.

Recall the double angle trigonometric formulae

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \quad \sin(A + B) = \cos A \sin B + \sin A \cos B.$$