

MATH 545 - MID-TERM EXAMINATION

All questions carry 10 marks.

The total mark available is 40; rescaling of the final mark may be carried out.

In the following questions, $\{Z_t\} \sim \text{WN}(0, \sigma_Z^2)$, with $t \in \mathbb{Z}$; ACVF refers to the *autocovariance* function. Here questions concerning stationarity refer to *weak* stationarity.

1. The time series process $\{X_t\}$ defined for all $t \in \mathbb{Z}$ by

$$X_t = U \cos(\omega t) + V \sin(\omega t) + Z_t$$

where $-\pi < \omega < \pi$ is a fixed parameter, U and V are uncorrelated random variables with expectation zero and variance one, and where U and V are uncorrelated with $\{Z_t\}$. Find, for each t , $\mathbb{E}[X_t X_{t+h}]$ for $h \geq 0$. Hence state whether $\{X_t\}$ is stationary. 10 MARKS

Hint: recall that

$$\cos(A - B) = \cos A \cos B + \sin A \sin B.$$

2. State, with justification, whether the following time series processes are stationary and causal:

- (a) $\{X_t\}$ defined implicitly for all t by the equation 5 MARKS

$$X_t = X_{t-1} - 0.24X_{t-2} + Z_t.$$

- (b) $\{X_t\}$ defined implicitly for all t by the equation 5 MARKS

$$X_t = 2X_{t-1} - X_{t-2} + Z_t.$$

3. Consider the stationary ARMA(1, 1) process

$$X_t = \frac{1}{2}X_{t-1} + Z_t + Z_{t-1}$$

State, with justification, whether this process is causal and invertible. 4 MARKS

Derive the linear process representation for $\{X_t\}$, that is, find the sequence $\{\psi_j\}$ such that

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}$$

is an infinite series representation of $\{X_t\}$. 6 MARKS

4. The autocovariance generating function (ACVGF) for a stationary process $\{X_t\}$ with ACVF $\gamma_X(h)$ is defined by

$$G_X(z) = \sum_{h=-\infty}^{\infty} \gamma_X(h) z^h \quad z \in \mathbb{C}$$

provided this sum is convergent for $1/r < |z| < r$ for some r . Compute the ACVGF for the MA(1) process $\{X_t\}$ defined by

$$X_t = Z_t - Z_{t-1}$$

by first computing the ACVF. 6 MARKS

Suppose $\{X_{1,t}\}$ and $\{X_{2,t}\}$ are two independent, zero mean, MA(1) processes. Is

$$X_t = X_{1,t} + X_{2,t}$$

also an MA(1) process? Justify your answer. 4 MARKS