

MATH 545: INTRODUCTION TO TIME SERIES ANALYSIS

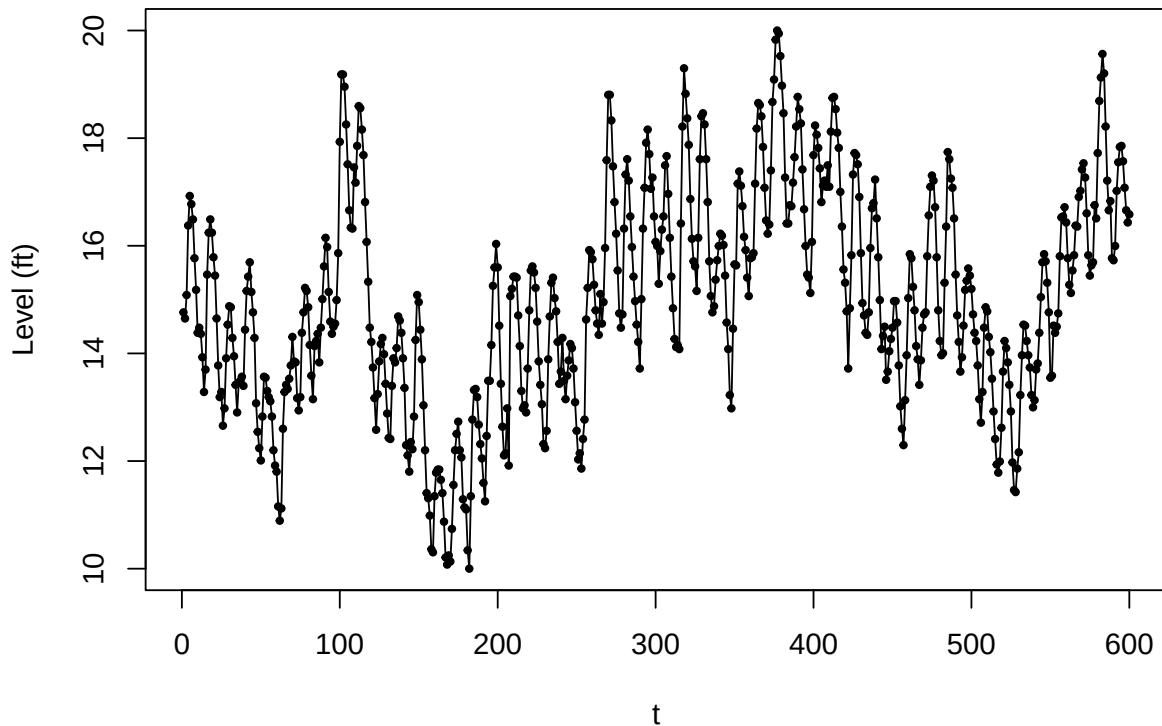
WINTER 2025

EXERCISES 3

Not for assessment.

1. The data set contained in the file `LakeErie.txt` is a collection of 600 monthly measurements of the level of Lake Erie, from January 1921 to December 1970.

```
erie.url<-'https://www.math.mcgill.ca/dstephens/545/Data/LakeErie.txt'  
erie<-scan(erie.url)  
par(mar=c(4,4,2,2))  
plot(erie,type='l',xlab='t',ylab='Level (ft)')  
points(erie,pch=19,cex=0.5)
```



- (a) Carry out an analysis of these data in R by examining ACF and PACF plots of the original and any appropriately differenced series.
- (b) Attempt to characterize the stochastic (random) component of the data using a SARIMA model.
- (c) Using the code provided in knitr sheets, formulate and fit a seasonal state space model to these data.

2. Consider the following state space model for scalar observations $Y_{1:n}$ and states $X_{1:n}$

$$\begin{aligned} Y_t &= \beta X_t + V_t \\ X_{t+1} &= \phi X_t + W_t \end{aligned}$$

where $\{V_t\}$ and $\{W_t\}$ are independent Gaussian white noise series with variances σ_V^2 and σ_W^2 respectively. To initiate the model, suppose that $X_1 \sim \text{Normal}(0, \sigma_W^2/(1 - \phi^2))$.

- Under what conditions is $\{Y_t\}$ stationary ?
- Assuming $\{Y_t\}$ is stationary, derive its ACVF, $\gamma_Y(h)$.
- Derive the Kalman filtering recursions for this model, based on the filtering and prediction distributions $p(x_t|y_{1:t})$ and $p(x_{t+1}|y_{1:t})$. Compute also the prediction distribution $p(y_{t+1}|y_{1:t})$.
- Using code similar to that provided in knitr sheets, or otherwise, perform maximum likelihood estimation of the system parameters for the data in the Ex3-Q2data.txt

<https://www.math.mcgill.ca/dstephens/545/Data/Ex3-Q2data.txt>

3. Formulate the stationary ARMA(2, 2) model

$$(1 - 0.5B)(1 - 0.9B)X_t = (1 + B)(1 + 2B)Z_t$$

as a state space model.

If it was required to add a **deterministic** seasonal component, $\{S_t\}$, with period 4 to this model, so that say

$$Y_t = X_t + S_t$$

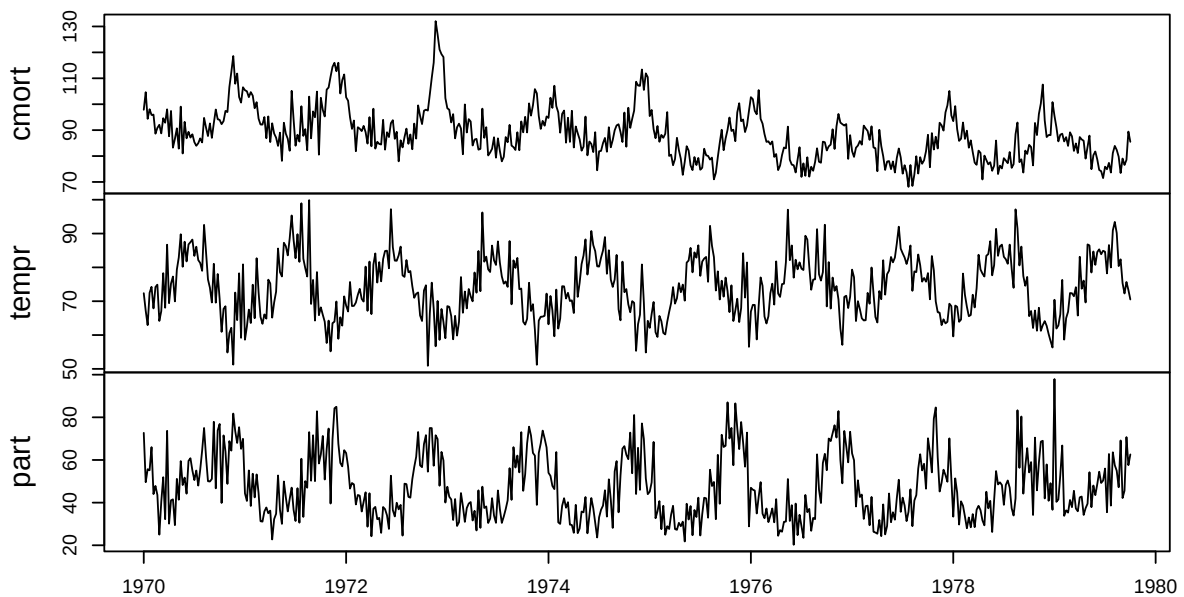
how would the state space model be amended? Identify the corresponding G and F matrices, and the other system components.

4. The following data from the `astsa` library comprise

- `cmort`: average weekly cardiovascular mortality in Los Angeles County, US. The data are 508 six-day averages obtained from daily values over the period 1970-1979;
- `tempr`: the corresponding weekly temperature data for the same period;
- `part`: a corresponding particulate matter series for the same period.

studied in the book by Shumway et al.

```
suppressMessages(library(vars));library(astsa)
x = cbind(cmort, tempr, part)
par(mar=c(4,4,0,2))
plot.ts(x, main = "", xlab = "")
```



- (a) Using the VAR function from the `vars` library, fit $\text{VAR}(p)$ models for various p , and compare them using the AIC.
- (b) Interpret the model selected using AIC, and check the fit of this model by inspecting the residual series.
- (c) Assess whether seasonal differencing of the series improves the quality of fit.