

MATH 545: INTRODUCTION TO TIME SERIES ANALYSIS
WINTER 2025

EXERCISES 1

Not for assessment.

In the following questions, $\{Z_t\} \sim \text{WN}(0, \sigma^2)$. The set $\mathbb{Z}$ is the set of integers

$$\mathbb{Z} \equiv \{\dots, -2, -1, 0, 1, 2, \dots\}.$$

1. Find the autocovariance function (ACVF), $\gamma_X(h)$, for the process $\{X_t\}$ defined by

$$X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2} \quad t \in \mathbb{Z}$$

Here, for $h = 0, 1, 2, \dots$,

$$\gamma_X(h) = \text{Cov}[X_t, X_{t+h}] = \mathbb{E}[X_t X_{t+h}] = \mathbb{E}[X_{t-h} X_t].$$

2. Identify whether each of the following numerical ACVFs, labelled (a), (b), (c) and (d),

h	$\gamma_X(h)$			
	(a)	(b)	(c)	(d)
0	1.0100	2.0200	1.21333	0.66667
1	-0.1000	-0.2000	0.50667	0.33333
2	0.0000	0.0000	0.25333	0.16667
3	0.0000	0.0000	0.12667	0.08333
4	0.0000	0.0000	0.06333	0.04167

correspond to

- a *moving average*, or MA(1), process,

$$X_t = Z_t + \theta_1 Z_{t-1}$$

for some θ_1 to be identified.

- an *autoregressive*, or AR(1) process.

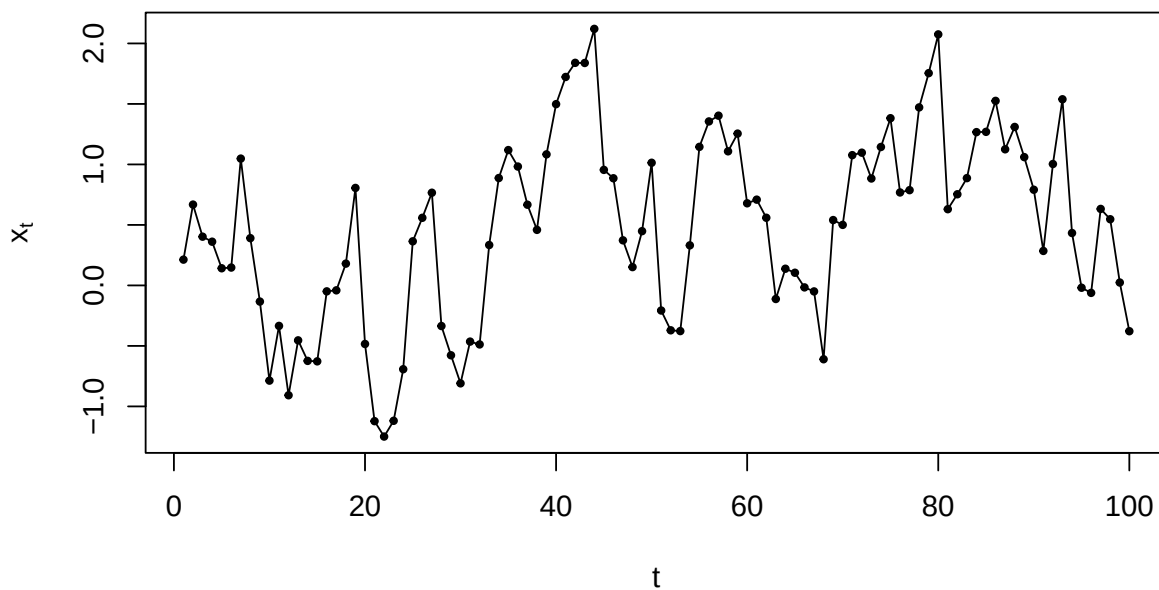
$$X_t = \phi_1 X_{t-1} + Z_t$$

for some ϕ_1 to be identified, where $|\phi_1| < 1$.

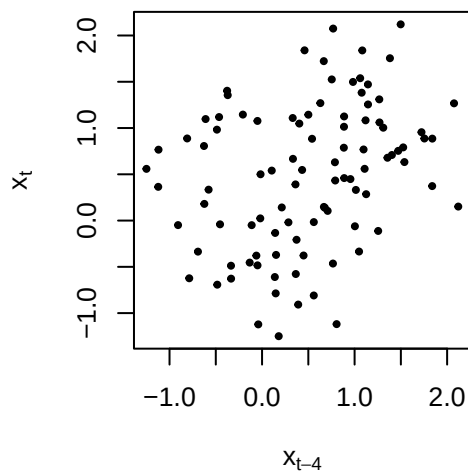
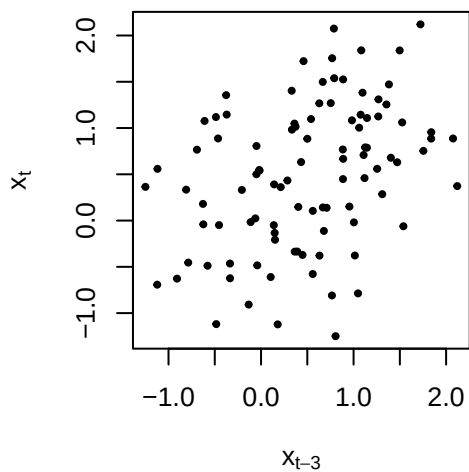
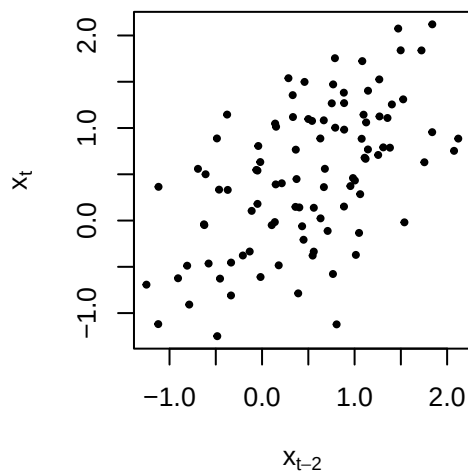
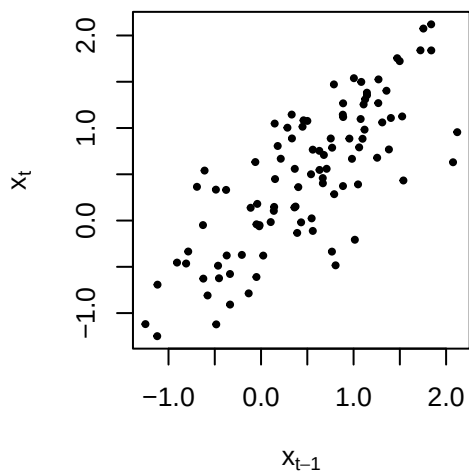
- some other process.

You may assume stationarity. The numerical values are rounded to four decimal places, and reported up to lag $h = 4$.

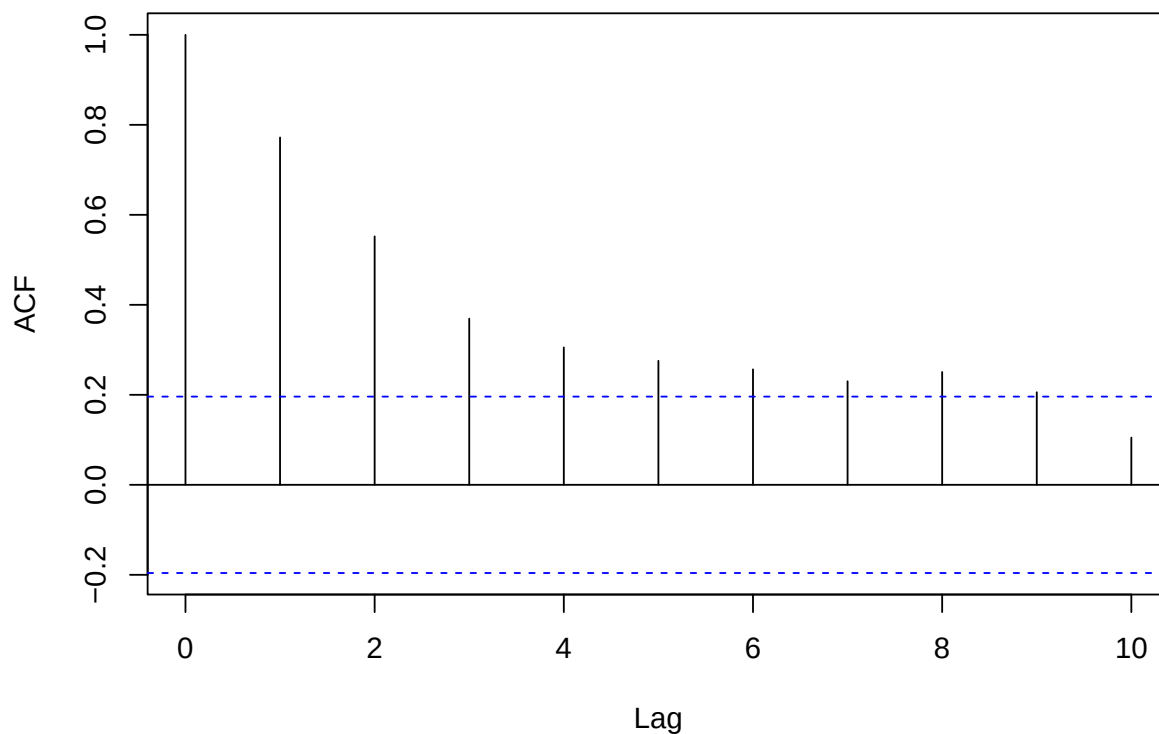
3. The following plots depict features of an observed time series, denoted $x_t, t = 1, \dots, 100$.



These scatterplots depict the relationship between x_t and x_{t-h} for $h = 1, 2, 3, 4$.



Finally, this is a plot of the **estimated** autocorrelation function (ACF), $\hat{\rho}_X(h)$, based on the 100 data points.



The blue dotted horizontal lines in this plot indicate statistical thresholds (i.e. *critical values*) for tests of the hypotheses

$$H_0 : \rho_X(h) = 0$$

$$H_a : \rho_X(h) \neq 0$$

for each h .

On the basis of this empirical information, suggest a time series model that may be appropriate as the data generating model for these data.