

MATH 545 - ASSIGNMENT 4

Please upload a single pdf with your solutions to myCourses by 11.59pm on Tuesday 25th March.

1. Compute and report the value of the AIC model selection criterion for the collection of ARMA(p, q) models for $0 \leq p, q \leq 2$ using the data set contained in the file

<https://www.math.mcgill.ca/dstephens/545/Assignments/A4/Q1.txt>

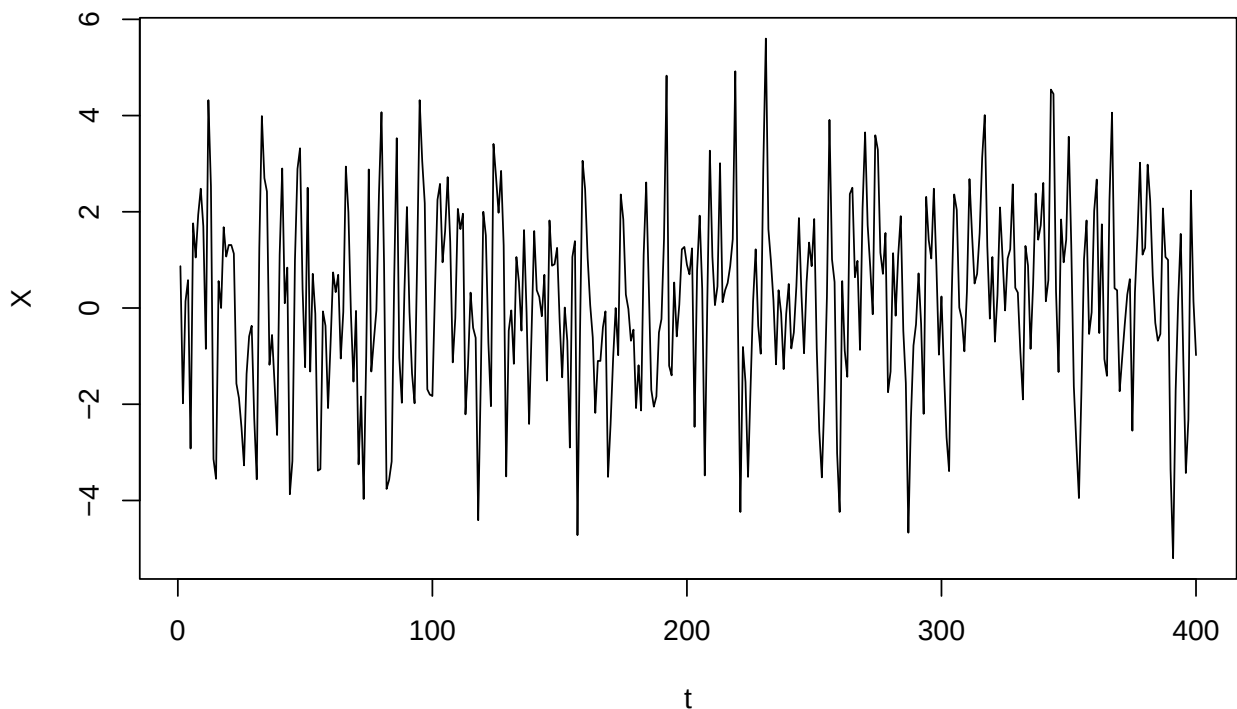
and hence report the selected model from those fitted.

4 Marks

```
file.url<-'https://www.math.mcgill.ca/dstephens/545/Assignments/A4/Q1.txt'
X<-scan(file.url)
n<-length(X)
n

+ [1] 400

par(mar=c(4, 4, 2, 0))
plot(X, type='l', xlab='t')
```



In R, the function `arima` may be used to compute the AIC quantity.

Inspect the sample ACF of the residuals for the two most favoured models under the AIC criterion, and hence comment on the suitability of these models for representing the observed time series.

2 Marks

2. Compute the first part of the autocovariance matrix sequence, $\Gamma_h, h = 0, 1, 2, 3$, for following bivariate processes $\{\mathbf{X}_t\}$ if, in each case, $\mathbf{Z}_t \sim \text{Normal}_2(\mathbf{0}, \mathbf{I}_2)$.

(a) The VAR(1) process

$$\mathbf{X}_t = \begin{bmatrix} 0.9 & 0.4 \\ 0.0 & 0.2 \end{bmatrix} \mathbf{X}_{t-1} + \mathbf{Z}_t$$

4 Marks

(b) The VMA(1) process

$$\mathbf{X}_t = \mathbf{Z}_t + \begin{bmatrix} 1.0 & -0.5 \\ 0.5 & -1.0 \end{bmatrix} \mathbf{Z}_{t-1}$$

4 Marks

(c) The VARMA(1, 1) process

$$\mathbf{X}_t = \begin{bmatrix} 0.5 & 0.5 \\ 0.0 & 0.5 \end{bmatrix} \mathbf{X}_{t-1} + \mathbf{Z}_t + \begin{bmatrix} 0.5 & 0.5 \\ 0.0 & 0.5 \end{bmatrix} \mathbf{Z}_{t-1}$$

6 Marks

You may use results from lectures to justify the formulae you use. You should provide numerical answers, giving justification for the calculation used.

Hint: For VARMA(p, q) process, the autocovariance matrix sequence can be computed by deriving the linear process-like representation: for example, the zero-mean, stationary and causal VARMA(p, q) process $\{\mathbf{X}_t\}$

$$\mathbf{X}_t - \Phi_1 \mathbf{X}_{t-1} - \dots - \Phi_p \mathbf{X}_{t-p} = \mathbf{Z}_t + \Theta_1 \mathbf{Z}_{t-1} + \dots + \Theta_q \mathbf{Z}_{t-q}$$

or, in the usual notation

$$\Phi(B)\mathbf{X}_t = \Theta(B)\mathbf{Z}_t$$

can be written

$$\mathbf{X}_t = \sum_{j=0}^{\infty} \Psi_j \mathbf{Z}_{t-j} = \Psi(B)\mathbf{Z}_t$$

where the matrices $\Psi_j, j = 0, 1, 2, \dots$ can be found via the identity

$$(\Psi_0 + \Psi_1 z + \dots + \Psi_j z^j + \dots) = (1 - \Phi_1 z - \dots - \Phi_p z^p)^{-1} (1 + \Theta_1 z + \dots + \Theta_q z^q).$$

Then, using methods identical to the univariate case, we can show that for $h > 0$

$$\Gamma_h = \mathbb{E}[\mathbf{X}_t \mathbf{X}_{t-h}^\top] = \sum_{j=0}^{\infty} \Psi_{j+h} \Sigma (\Psi_j)^\top.$$