

MATH 545 - ASSIGNMENT 3

*Please upload a single pdf with your solutions to myCourses by
11.59pm on Friday 28th February.*

In the following questions, $\{Z_t\} \sim \text{WN}(0, \sigma_Z^2)$, with $t \in \mathbb{Z}$. In computing the numerical results, you may use pre-written R functions such as `ARMAacf`.

1. Compute, using R or otherwise, the optimal linear one-step-ahead forecast for X_{n+1} based the following data set of $n = 11$ data points

-0.93 -0.89 -0.63 -0.38 0.76 0.03 -0.56 -0.19 -0.11 -0.82 -0.49

assuming the following models:

- (a) The AR(1) model: $X_t = 0.9X_{t-1} + Z_t$; 2 MARKS
- (b) The ARMA(1, 1) model: $X_t = 0.5X_{t-1} + Z_t - Z_{t-1}$; 3 MARKS
- (c) The ARMA(2, 1) model: $X_t = 0.2X_{t-1} - 0.5X_{t-2} + Z_t + Z_{t-1}$. 3 MARKS

Give the numerical value of the forecast, and the code used. Here, it is not necessary to use the recursive algorithms for matrix inversion as the matrix inversion function `solve` (or equivalent) can be used directly.

2. Consider the model for stationary process $\{X_t\}$ defined by

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_{12} Z_{t-12}. \quad (1)$$

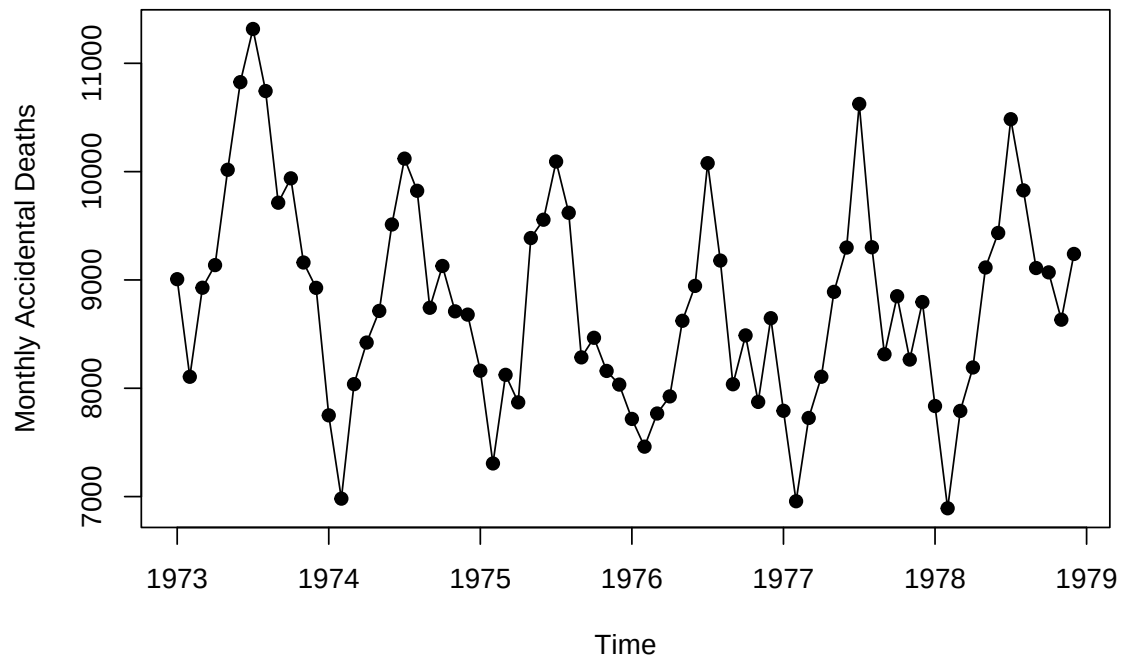
- (a) Compute the form of the ACVF, $\gamma_X(h)$. 4 MARKS
- (b) The data `USAccDeaths` available in R below are stationary in mean (mean estimated to be 8788.792, with no trend), but also exhibit periodic and non-stationary behaviour. After differencing successively at lag 1 and lag 12 (using the difference operators ∇ then ∇_{12}), the periodic and non-stationary behaviour can be removed. That is, denoting the original data process $\{Y_t\}$ and the differenced data process $\{X_t\}$, we define

$$X_t = \nabla_{12} \nabla Y_t = (1 - B^{12})(1 - B)Y_t$$

and study $\{X_t\}$.

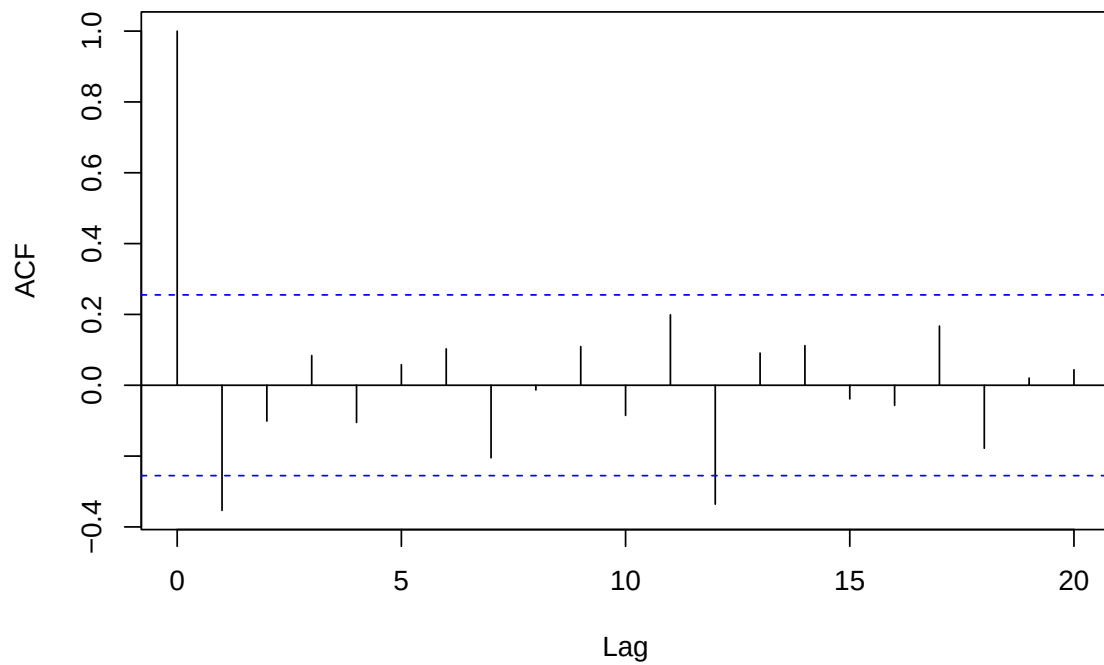
```
par(mar=c(4, 4, 3, 2))
plot(USAccDeaths, ylab="Monthly Accidental Deaths")
points(USAccDeaths, pch=19)
title("Accidental Deaths: Monthly totals \n Jan 1973 - Dec 1978")
```

Accidental Deaths: Monthly totals Jan 1973 – Dec 1978

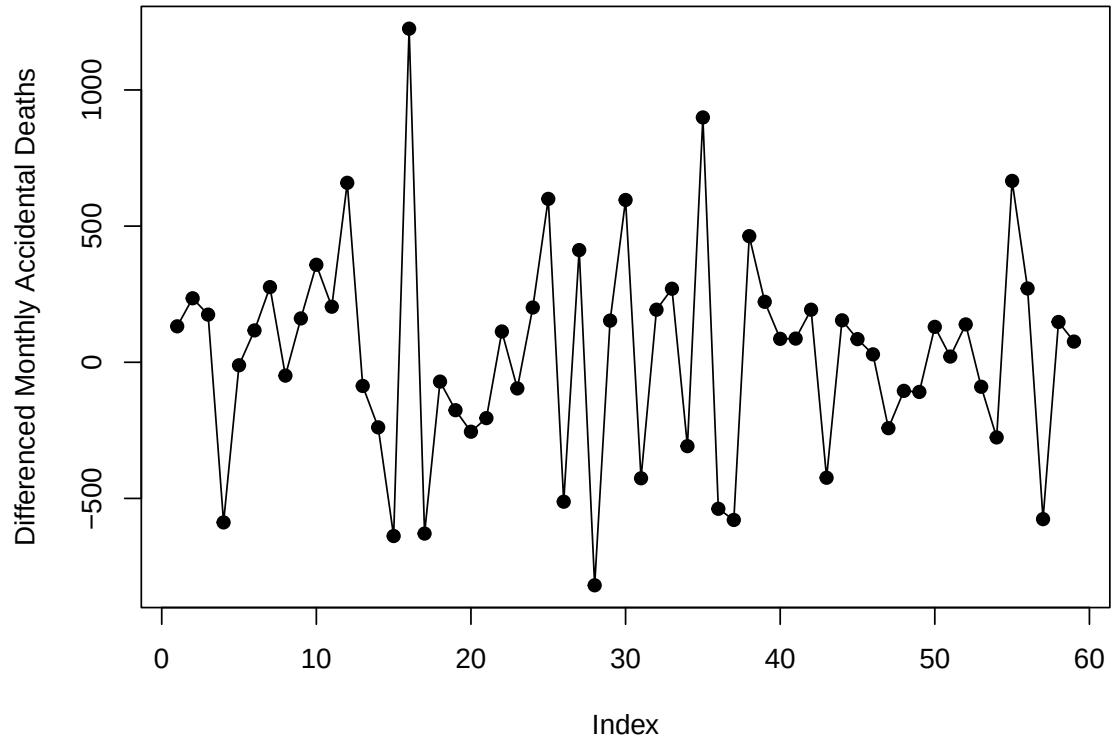


```
Yt<-as.numeric(USAccDeaths);Xt<-diff(diff(Yt),lag=12)
par(mar=c(4,4,3,2))
acf(Xt,lag.max=20,main='ACF of Xt')
```

ACF of Xt



```
par(mar=c(4,4,3,2))
plot(Xt,type='l',ylab="Differenced Monthly Accidental Deaths")
points(Xt,pch=19)
```



Using the differenced data X_t , estimate the parameters θ_1 and θ_{12} in the model (1) when used for these data by equating theoretical and sample values of the ACF.

6 MARKS

- (c) Explain how to compute the one-step-ahead forecasts for $\{X_t\}$ and $\{Y_t\}$ based on the available data (you do not need to compute the forecasts).

2 MARKS