

MATH 545 - ASSIGNMENT 2

Please upload a single pdf with your solutions to myCourses by 11.59pm on Tuesday 11th February.

1. For each of the following stationary processes, $\{X_t\}$, defined via their model equations in terms of $\{Z_t\} \sim \text{WN}(0, 1)$, find the corresponding linear process representation

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}$$

that is, identify the sequence $\{\psi_j\}$. Show your derivation. If no such linear process representation exists, explain why.

(a) $X_t = Z_t + Z_{t-1} + Z_{t-4}$. 2 MARKS

(b) $X_t = \frac{1}{4}X_{t-2} + Z_t$. 3 MARKS

(c) $X_t = \frac{3}{4}X_{t-1} + Z_t + \frac{1}{2}Z_{t-1}$. 3 MARKS

2. State, with justification, whether the AR(2) process defined in terms of $\{Z_t\} \sim \text{WN}(0, 1)$ via the equation

$$X_t = X_{t-1} - \frac{1}{2}X_{t-2} + Z_t \quad t \in \mathbb{Z}$$

is stationary and causal. 4 MARKS

3. Suppose that $\{X_t\}$ is weakly stationary. Is the process $\{Y_t\}$ defined by

$$Y_t = X_{-t} \quad t \in \mathbb{Z}$$

weakly stationary? Justify your answer. 4 MARKS

4. Suppose that $\{X_{1,t}\}$ and $\{X_{2,t}\}$ are independent processes (that is, $X_{1,t}$ and $X_{2,s}$ are independent for all t and s), with

$$X_{1,t} \sim \text{AR}(1) \quad X_{2,t} \sim \text{AR}(1)$$

defined with respect to two independent white noise processes $\{Z_{1,t}\} \sim \text{WN}(0, 1)$ and $\{Z_{2,t}\} \sim \text{WN}(0, 1)$. Suppose specifically that

$$X_{1,t} = \phi X_{1,t-1} + Z_{1,t}$$

$$X_{2,t} = \varphi X_{2,t-1} + Z_{2,t}$$

where $0 < \phi, \varphi < 1$, with the usual assumption that

$$\mathbb{E}[X_{1,t}Z_{1,s}] = \mathbb{E}[X_{2,t}Z_{2,s}] = 0 \quad s > t.$$

Is the process $\{Y_t\}$ defined by

$$Y_t = X_{1,t} + X_{2,t} \quad t \in \mathbb{Z}$$

stationary and causal? Justify your answer. 4 MARKS