

MATH 545 - ASSIGNMENT 1

Please upload a single pdf with your solutions to myCourses by 11.59pm on Sunday 26th January.

1. For each of the following time series processes, $\{X_t\}$, derive the ACVF, $\gamma_X(h)$, $h = 0, \pm 1, \pm 2, \dots$ from the model equation. In each case, $\{Z_t\} \sim \text{WN}(0, 1)$.

- (a) $X_t = Z_t - Z_{t-1}$. 3 MARKS
- (b) $X_t = Z_t - Z_{t-2}$. 3 MARKS
- (c) $X_t = \nabla^2 Z_t = (1 - B)^2 Z_t$. 3 MARKS
- (d) $X_t = \lambda^2 X_{t-2} + Z_t$, for constant λ where $-1 < \lambda < 1$. 3 MARKS

Assume that $\{X_t\}$ is stationary in all cases. Show your working.

2. For a stationary AR(1) process, we can write the model equation

$$X_t - \phi_1 X_{t-1} = Z_t \quad t \in \mathbb{Z}$$

or equivalently using the backshift operator

$$(1 - \phi_1 B)X_t = Z_t$$

where we assume $|\phi_1| < 1$.

Using the *inverse operator* expression $(1 - \phi_1 B)^{-1}$, and the identity

$$(1 - \phi_1 B)^{-1}(1 - \phi_1 B)X_t = X_t \quad (1)$$

verify that the inverse operator can be expressed as

$$(1 - \phi_1 B)^{-1} = \sum_{j=0}^{\infty} \phi_1^j B^j \quad (2)$$

and hence that we may write

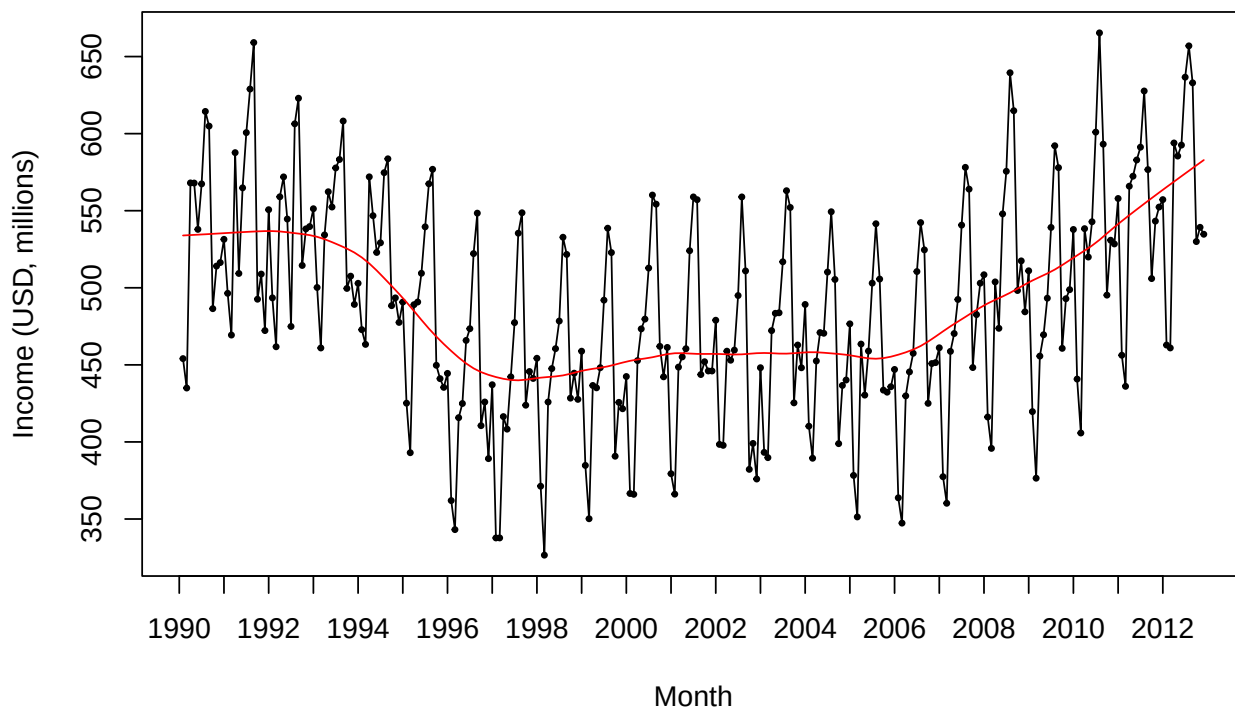
$$X_t = \sum_{j=0}^{\infty} \phi_1^j Z_{t-j}$$

as given by the recursion from lectures. Specifically, verify that identity (1) holds for the expansion claimed in (2). 5 MARKS

3. The following 275 data points represent monthly income (in millions of US dollars) from rail passengers on a national rail carrier between 1990 and 2012. In this data set, some pre-processing is necessary to account for external economic factors; for example, the `lowess` smoothing function may be used.

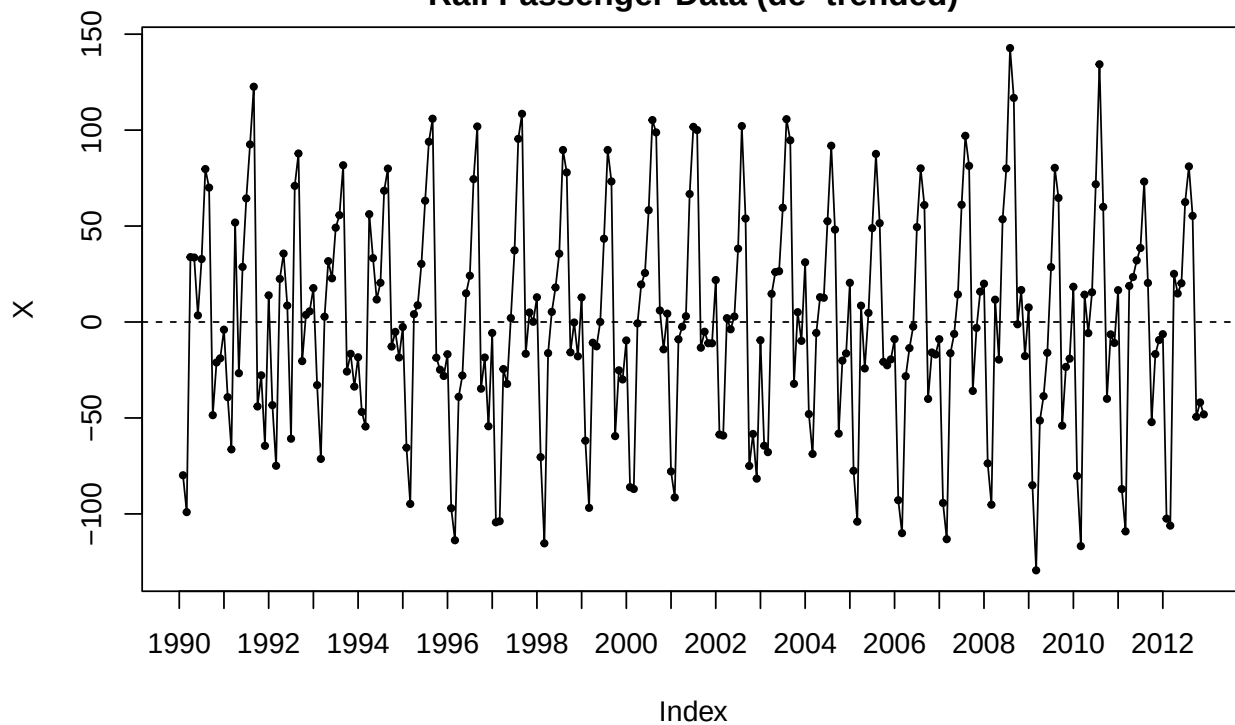
```
file.url<-'https://www.math.mcgill.ca/dstephens/545/Assignments/A1/RailPass.txt'
Rail<-scan(file.url)
par(mar=c(4, 4, 2, 0))
smooth.mu<-lowess(Rail, f=1/5) #Smoothing by local linear fitting
X<-Rail-smooth.mu$y
plot(Rail, pch=19, cex=0.4, xlab='Month', ylab='Income (USD, millions)', xaxt='n')
axis(1, at=c(0:22)*12, seq(1990, 2012, by=1))
lines(Rail); title('Rail Passenger Data')
lines(smooth.mu, col='red')
```

Rail Passenger Data



```
plot(X,pch=19,cex=0.5, xaxt='n'); lines(X); abline(h=0,lty=2)
title('Rail Passenger Data (de-trended)')
axis(1,at=c(0:22)*12,seq(1990,2012,by=1))
```

Rail Passenger Data (de-trended)



Using software, compute and plot the estimated ACF for the de-trended series to lag 24, and comment on its features. 3 MARKS