

MATH 545 - ASSIGNMENT 1

*Please upload a pdf with your solutions to myCourses by
10.00pm on Friday 29th September.*

In the following questions, $\{Z_t\} \sim \text{WN}(0, \sigma^2)$, with $t \in \mathbb{Z}$, that is, we consider the process defined for all integers

- 1 Using the techniques established for the autoregressive model of order 1 (AR(1)), compute the autocorrelation function, $\rho_X(h)$, $h = 0, \pm 1, \pm 2, \dots$ for the following stationary processes:

- (a) $\{X_t\}$ defined implicitly for all t by the equation

$$X_t + 0.1X_{t-1} = Z_t$$

2 MARKS

- (b) $\{X_t\}$ defined implicitly for all t by the equation

$$X_t = 1.1X_{t-1} - 0.3X_{t-2} + Z_t$$

Hint: first find the representation

$$X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j} \quad t \in \mathbb{Z}$$

for constants ψ_j , $j = 0, 1, 2, \dots$ using the recursive definition.

4 MARKS

- (c) $\{X_t\}$ defined explicitly for all t by the equation

$$X_t = Z_t + Z_{t-1} - 0.5Z_{t-2}$$

3 MARKS

In each case, show the details of the derivation.

- 2 The *backshift* operator, B , is an operator that acts on time series processes $\{X_t\}$ such that for every t , $BX_t = X_{t-1}$. The operator has standard algebraic and arithmetic properties such that, for example, we can form polynomials in B : if $\Phi(B)$ is such a polynomial of order $p \geq 1$, then

$$\Phi(B)X_t \equiv \left(\sum_{j=0}^p \phi_j B^j \right) X_t = (\phi_0 + \phi_1 B + \phi_2 B^2 + \dots + \phi_p B^p) X_t \equiv \sum_{j=0}^p \phi_j X_{t-j}.$$

for constants $\phi_0, \phi_1, \dots, \phi_p$.

Show that the process $\{X_t\}$ from 1 (b) can be written in the form

$$\Phi(B)X_t = Z_t$$

where $\Phi(z) = 1 + \phi_1 z + \phi_2 z^2$ is a polynomial of order two. Show that

$$\Phi(B) = (1 - \alpha_1 B)(1 - \alpha_2 B)$$

for two possibly complex values α_1, α_2 .

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- 3 Using R, generate and plot realizations of length $n = 200$ from each of the processes in question 1, assuming that $Z_t \sim \text{Normal}(0, 1)$. Show the code you use, and provide the resulting plots.

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