MATH 523: RESIDUALS

Three types of residual are used for checking the fit of a GLM

- **Pearson residual**

$$r_{Pi} = \frac{y_i - \widehat{\mu}_i}{\sqrt{V(\widehat{\mu}_i)}}$$

- **Anscombe residual**

$$r_{Ai} = \frac{A(y_i) - A(\widehat{\mu}_i)}{\dot{A}(\widehat{\mu}_i)\sqrt{V(\widehat{\mu}_i)}}$$

where function A is defined by

$$A(t) = \int_{-\infty}^{t} \frac{ds}{V^{1/3}(s)}$$

The function $A(.)$ is chosen such that, for models in the Exponential/Exponential-Dispersion family, the skewness of the distribution of the transformed variable $A(Y)$ is as small as possible.

- **Deviance Residual**

$$D(\boldsymbol{y}; \widehat{\theta}) = \frac{1}{\phi}\sum_{i=1}^{n} D_i(y_i; \widehat{\theta}) = \frac{1}{\phi}\sum_{i=1}^{n} D_i$$

say, where

$$D_i = 2w_i\left\{y_i(\widetilde{\theta} - \widehat{\theta}_i) - (b(\widetilde{\theta}_i) - b(\widehat{\theta}_i))\right\}$$

Then the deviance residual is

$$r_{Di} = \text{sign}(y_i - \widehat{\mu}_i)\sqrt{D_i} = \begin{cases} \sqrt{D_i} & y_i > \widehat{\mu}_i \\ -\sqrt{D_i} & y_i < \widehat{\mu}_i \end{cases}$$

In the Poisson case

$$\begin{aligned} r_{Pi} &= \frac{y_i - \widehat{\mu}_i}{\sqrt{\widehat{\mu}_i}} \\ r_{Ai} &= \frac{3(y_i^{2/3} - \widehat{\mu}_i^{2/3})}{2\widehat{\mu}_i^{1/6}} \\ r_{Di} &= \text{sign}(y_i - \widehat{\mu}_i)\left\{2(y_i \log\frac{y_i}{\widehat{\mu}_i} - y_i + \widehat{\mu}_i)\right\}^{1/2} \end{aligned}$$

where $\widehat{\mu}_i = \widehat{\lambda}_i = \exp\{x_i\beta\}$

```
f0<-glm(Y~1,family=poisson)
f1<-glm(Y~X,family=poisson)

rp0<-residuals(f0,type="pearson")
rd0<-residuals(f0,type="deviance")
fA0<-fitted(f0)
rA0<-1.5*(Y^(2/3)-fA0^(2/3))/fA0^(1/6)
rp1<-residuals(f1,type="pearson")
rd1<-residuals(f1,type="deviance")
fA1<-fitted(f1)
rA1<-1.5*(Y^(2/3)-fA1^(2/3))/fA1^(1/6)
```

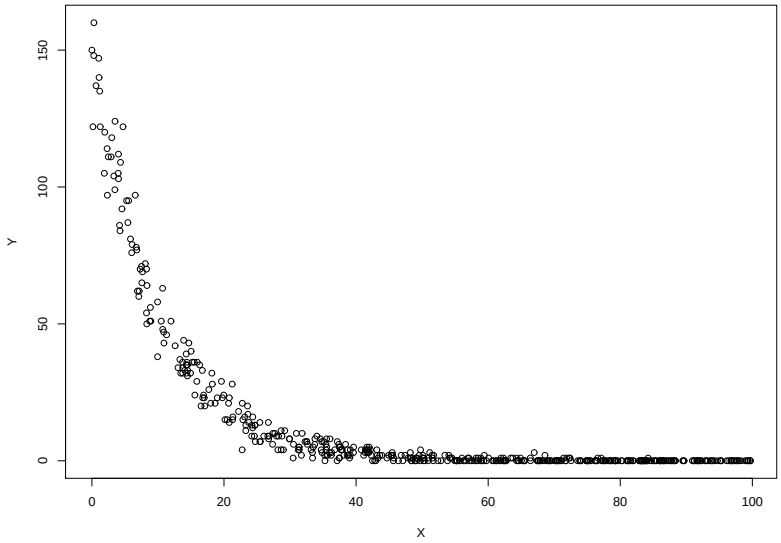


Figure 1: Poisson data Y with single covariate X

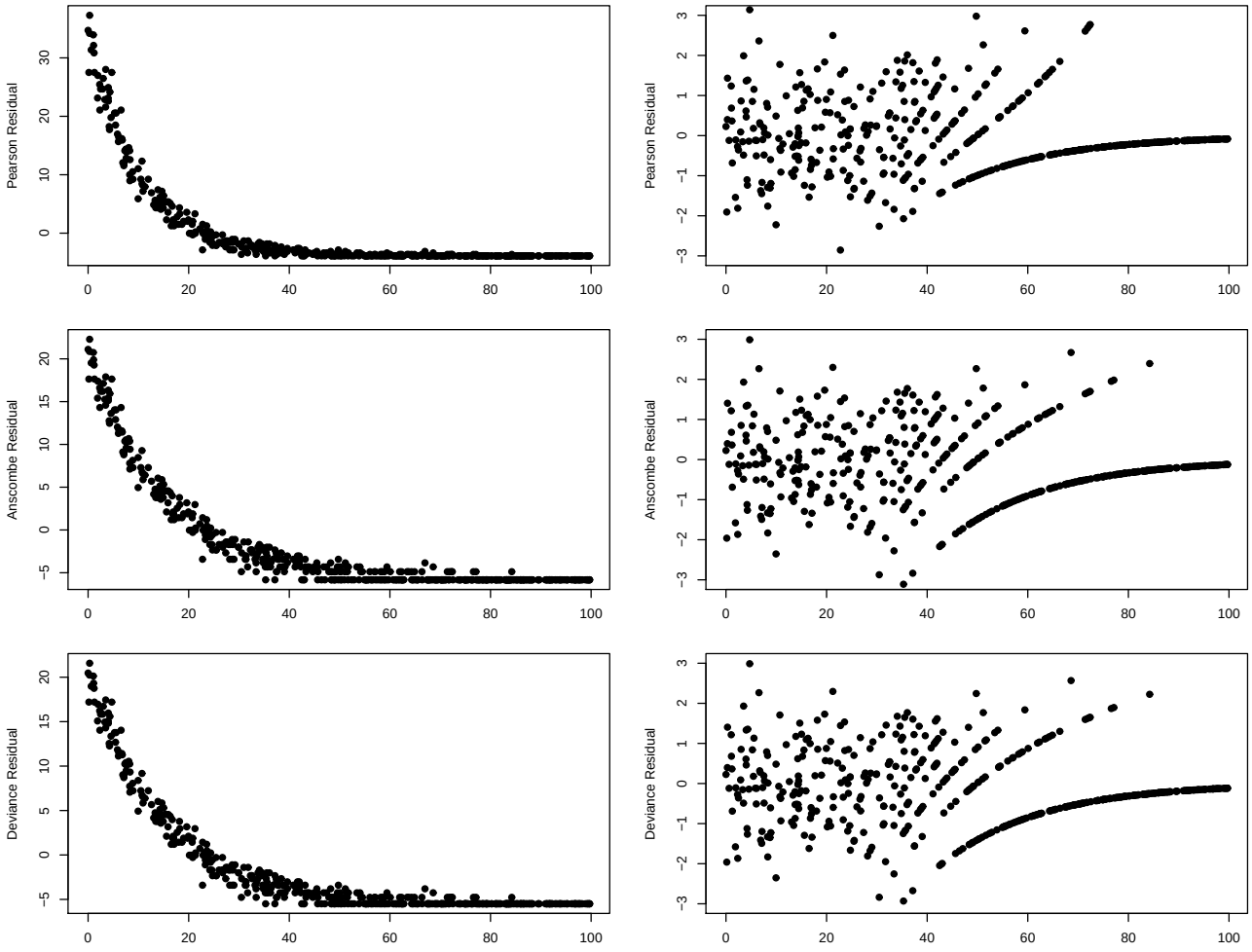


Figure 2: Residuals from fits of null (left column) and single predictor (right column): Pearson (top), Anscombe (middle) and deviance (bottom) residuals.