

MATH 523 - EXERCISES 2: SOLUTIONS

- 1 Let D , R and A denote the factors Deviant, At Risk and Adversity respectively. The list of model fit details is as follows

Model	Deviance	Res. Df.
0 1	100.718	11
1 D	56.288	10
2 R	98.389	10
3 A	63.178	9
4 $D + R$	53.959	9
5 $D + A$	18.748	8
6 $R + A$	60.849	8
7 $D + R + D.R$	52.517	8
8 $D + A + D.A$	15.092	6
9 $R + A + R.A$	49.990	6
10 $D + R + A$	16.419	7
11 $D + R + A + D.R$	14.977	6
12 $D + R + A + D.A$	12.763	5
13 $D + R + A + R.A$	5.560	5
14 $D + R + A + D.R + D.A$	11.321	4
15 $D + R + A + D.R + R.A$	4.118	4
16 $D + R + A + D.A + R.A$	1.904	3
17 $D + R + A + D.R + D.A + R.A$	0.943	2
18 $A * B * C$	0.000	0

	0.95 quantiles of the Chi-squared distribution for degrees of freedom ν										
ν	1	2	3	4	5	6	7	8	9	10	11
$\chi^2_{\nu}(0.95)$	3.841	5.991	7.815	9.488	11.07	12.592	14.067	15.507	16.919	18.307	19.675

- 17 vs 18: $\Delta\text{Dev.} = 0.943$, $\Delta\text{df} = 2$ - do not reject 17 as an adequate simplification of 18.
- Attempt to drop one two-way interaction:
 - 17 vs 14 $\Delta\text{Dev.} = 10.378$, $\Delta\text{df} = 2$ - reject 14 as an adequate simplification of 17.
 - 17 vs 15 $\Delta\text{Dev.} = 3.175$, $\Delta\text{df} = 2$ - do not reject 15 as an adequate simplification of 17.
 - 17 vs 16 $\Delta\text{Dev.} = 0.961$, $\Delta\text{df} = 1$ - do not reject 16 as an adequate simplification of 17.

From this sequence we realize that we cannot drop $R.A$, but possibly can drop one of the other two-way interactions.

- Removing $D.R$ and $D.A$ terms:
 - Attempt to drop $D.R$ from 15 (ie 15 vs 13): $\Delta\text{Dev.} = 1.442$ $\Delta\text{df} = 1$ - do not reject 13 as an adequate simplification of 15.
 - Attempt to drop $D.A$ from 16 (ie 16 vs 13): $\Delta\text{Dev.} = 3.656$ $\Delta\text{df} = 2$ - do not reject 13 as an adequate simplification of 16.

From this we conclude that 13 is an adequate one interaction simplification: notice that the comparison with 18 could have been made directly (18 vs 13: $\Delta\text{Dev.} = 5.560$ $\Delta\text{df} = 5$ - do not reject 13 as an adequate simplification of 18).

- Attempt to remove the remaining interaction (13 vs 10): $\Delta\text{Dev.} = 10.859$ $\Delta\text{df} = 2$ - reject 10 as an adequate simplification of 13.
- Attempt to remove D (13 vs 9): $\Delta\text{Dev.} = 44.430$ $\Delta\text{df} = 1$ - reject 9 as an adequate simplification of 13.

The conclusion is that the final model is $D + R + A + R.A \equiv D + R * A$. This model implies “partial independence” between D and (R, A) . With deviance 5.560 on 5 degrees of freedom, the model fit is adequate, and there is no evidence of overdispersion.

2 The null deviance for this data set is 489.6 on 31 degrees of freedom. We start by trying to simplify the saturated model by dropping interactions:

- Dropping the three way interaction `hair:eye:sex` yields $\Delta\text{Dev.} = 8.187$ $\Delta\text{df} = 9$. We therefore do obtain an adequate simplification by removing that term.
- Attempting to drop one of the two-way interactions does not yield an adequate simplification
 - Remove `hair:eye` $\Delta\text{Dev.} = 154.02$, $\Delta\text{df} = 9$.
 - Remove `hair:sex` $\Delta\text{Dev.} = 14.896$ $\Delta\text{df} = 3$.
 - Remove `eye:sex` $\Delta\text{Dev.} = 13.845$ $\Delta\text{df} = 3$.
- The model with three-way interaction removed cannot be simplified further with significant increase in deviance.
- The model that includes `sex` does fit significantly better than the model that excludes it (fitted earlier in class).

3 The following results are obtained for the fits of the models (p is the number of parameters fitted):

Model	Deviance	Res. Df.	p
Independence	125923	9	7
Quasi-Independence	69.51	5	11
Symmetry	243.55	6	10
Quasi-symmetry	2.98	3	13

- (a) By an inspection of deviance figures and residual degrees of freedom, the only model for which the Poisson assumption appears to be valid is the quasi-symmetry model.
- (b) The fitted values are as follows

	Independence				Quasi-independence			
	NE	MW	S	W	NE	MW	S	W
NE	2599.06	3089.07	4136.62	2372.25	11607.00	126.56	312.91	150.53
MW	3107.07	3692.85	4945.16	2835.93	117.39	13677.00	531.11	255.50
S	3939.18	4681.85	6269.54	3595.43	133.23	243.80	17819.00	289.97
W	2283.69	2714.24	3634.68	2084.40	71.39	130.64	322.98	10192.00

	Symmetry				Quasi-symmetry			
	NE	MW	S	W	NE	MW	S	W
NE	11607.00	93.50	269.00	93.50	11607.00	95.79	370.44	123.77
MW	93.50	13677.00	370.00	239.00	91.21	13677.00	501.68	311.11
S	269.00	370.00	17819.00	278.00	167.56	238.32	17819.00	261.12
W	93.50	239.00	278.00	10192.00	63.23	166.89	294.88	10192.00

- (c) As explained in lectures, the symmetry model imposes *marginal homogeneity* (that is, identical marginal distributions for each factor), whereas the quasi-symmetry model does not.