MATH 323 - SKILL TEST 1

1. Four letters are to be selected from the first eight letters of the alphabet, abcdefgh. How many distinct letter patterns can be formed if the selection is made **without replacement** ?

   Solution: by the multiplication principle, the number is

   $$8 \times 7 \times 6 \times 5 = P_4^8 = 1680.$$

2. In a multiple choice test, each question has five possible answers: A,B,C,D,E. How many possible answer patterns are there, if it is known that each answer letter must appear **precisely twice** in the test ?

   Solution: consider selecting positions for the letters to appear in the answer sequence: by the multiplication principle, we have that the number of patterns is

   $$\binom{10}{2} \times \binom{8}{2} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{2} = \frac{10!}{2! \times 2! \times 2! \times 2! \times 2!} = 113400.$$

   For the As, we must choose 2 positions from 10 available; having chosen those, for the Bs, we must choose 2 positions from the 8 remaining etc.

3. A bag contains four Red balls, three Blue balls, two Green balls and one Yellow ball. Balls are to be drawn from the bag, with each ball equally likely to be selected from those remaining on each draw.

   What is the probability that the Yellow ball is not selected on any of the first ten draws, if draws are made with replacement ? Enter the answer to four decimal places in Blank 1 (eg 0.1234).

   What is the probability that precisely two Red balls are selected in the first five draws, if the draws are made without replacement ? Enter the answer to four decimal places in Blank 2 (eg 0.1234).

   Solution: For the first part, drawing with replacement, each ball is equally likely at each draw, so the probability is

   $$\frac{\text{\# draw sequences containing no Yellow ball}}{\text{\# draw sequences of any configuration}} = \frac{9 \times 9 \times \cdots \times 9}{10 \times 10 \times \cdots \times 10} = \left(\frac{9}{10}\right)^{10} = 0.3487.$$

   For the second part, we use the hypergeometric formula with $(N, R, n, r) = (10, 4, 5, 2)$ as we are drawing a sample of size 5 from the population of size 10, with our focus on the 2 Red balls that are included in the sample. The probability is

   $$\frac{\binom{n}{r}\binom{N-n}{R-r}}{\binom{N}{R}} = \frac{\binom{5}{2}\binom{5}{2}}{\binom{10}{4}} = 0.4762.$$

4. Three committees are to be constructed from a list of 12 candidates. One committee requires six members, the other two committees each require three members. How many distinct committees can be created ?

   Solution: This requires an application of the partitioning principle and multinomial coefficients

   $$\frac{12!}{6! \times 3! \times 3!} = 18480.$$