# MATH 323 - EXERCISES 8

## NOT FOR ASSESSMENT.

## TRANSFORMATIONS

1. Suppose that $Y$ is a continuous random variable with range $\mathcal{Y} = [0, 1]$, and pdf $f_Y$ specified by

$$f_Y(y) = 2(1-y) \qquad 0 \le y \le 1$$

and zero otherwise. Find the probability distributions of random variables $U_1$, $U_2$ and $U_3$ defined respectively by

$$U_1 = 2Y - 1 \qquad U_2 = 1 - 2Y \qquad U_3 = Y^2$$

In each case, find the range of values that the new variable can take, and its pdf or cdf.

2. The annual profit (in millions of dollars) of a manufacturing company is a function of product demand. If $Y$ is the continuous random variable corresponding to the demand in a given year, then the annual profit is also a continuous random variable, $U$ say, where $U = 2(1 - e^{-2Y})$.

If $Y$ has an Exponential distribution with parameter $\beta = 1/6$, find the expected annual profit.

3. The random variable $Y$ has a continuous uniform distribution on the interval $[-1, 1]$. Find the pdfs of the transformed random variables

$$U = |Y| \qquad Z = Y^2$$

4. If $Y$ is **any** continuous random variable with distribution function $F_Y$, show that

   (a) $U = F_Y(Y)$ has a continuous Uniform distribution on $[0, 1]$, and
   (b) $Z = -\ln U$ has an Exponential distribution.

5. If $Y$ is a continuous random variable with pdf specified by

$$f_Y(y) = \alpha\beta y^{\alpha-1} e^{-\beta y^{\alpha}} \qquad y > 0$$

and zero otherwise, for parameters $\alpha, \beta > 0$, then $Y$ has a *Weibull* distribution. Show that $U = Y^{\alpha}$ has an Exponential distribution.

6. Suppose that random variable $Y$ has a standard normal distribution.

   (a) Find the cdf of random variable $X = Y^2$ in terms of the standard normal cdf, denoted $\Phi$. Recall that for the cdf of $X$, we have

$$P(X \le x) = P(Y^2 \le x) = P(|Y| \le \sqrt{x})$$

   (b) Find the pdf of $X$, $f_X$.
   (c) Identify (by name) the probability distribution of $X$.

7. Suppose that $Y_1$ and $Y_2$ are independent and identically distributed random variables, each having a standard normal distribution. Let random variable $V$ be defined by

$$V = Y_1^2 + Y_2^2$$

Find the mgf and hence the pdf of $V$.