MATH 323 - EXERCISES 7

NOT FOR ASSESSMENT.

CONTINUOUS PROBABILITY DISTRIBUTIONS

1. The length of time (in hours) that a student takes to complete a one hour exam is a continuous random variable Y with probability density function f defined by

$$f(y) = cy^2 + y \qquad 0 < y \leq 1$$

for some constant c, and zero otherwise.

(a) Find the value of c.

(b) By integration, find the cumulative distribution function of Y, F

(c) Find the probability that a student completes the exam in less than half an hour.

(d) **Given** that a student takes longer than fifteen minutes to complete the exam, find the probability that they require at least half an hour, that is, find the conditional probability

$$P\left(Y > \frac{1}{2} \,\middle|\, Y > \frac{1}{4} \right)$$

(e) In a class of two hundred students, find the probability that at most three students complete the exam in fewer than ten minutes, if the exam completion times for the two hundred students are independent random variables having the distribution specified above.

Hint: Consider independent discrete random variables $Y_1, ...Y_{200}$ where

$$Y_i = \begin{cases} 1 & \textit{student } i \textit{ completes the exam in fewer than ten minutes} \\ 0 & \textit{otherwise} \end{cases}$$

2. A continuous random variable Y has probability density function f specified by

$$f(y) = c(1 - y^2) \qquad -1 \leq y \leq 1$$

for some constant c, and zero otherwise

Determine the constant c, and obtain the cumulative distribution function of Y, F.

3. Consider a triangle with two sides of unit length. The angle between these two sides is a continuous random variable Y with probability density function f specified by

$$f(y) = cy(\pi - y) \qquad 0 < y < \pi/2$$

for some constant c, and zero otherwise.

(a) Find the value of the constant c.

(b) Let X be the continuous random variable corresponding to the area of this triangle. Find the range, $\mathcal{X}$, and an expression for the probability density function of X. *(Hint: first compute cdf of X in terms of the cdf of Y).*

4. Find the probability density function of continuous random variable Y whose range is $\mathcal{Y} = \mathbb{R}^+ = \{y : y \geq 0\}$ and whose cumulative distribution function is specified as

$$F(y) = 1 - e^{-y^2} \qquad y \geq 0$$

5. The probability density function of continuous random variable Y taking values on range $\mathcal{Y} = (0, 2)$ is specified by

$$f(y) = \begin{cases} y & 0 < y < 1 \\ 2 - y & 1 \leq y < 2 \end{cases}$$

and zero otherwise. Find the cumulative distribution function of Y, F, and hence find

$$P(\, 0.8 < Y \leq 1.2 \,)$$

6. Suppose that there are two mutually exclusive causes of machine failure, A and B, with which occur with probabilities 0.3 and 0.7 respectively. If the cause of failure is A, the repair-time random variable, T, has a Normal distribution with parameters $\mu_A = 2$ hours and $\sigma = 45$ minutes, whereas if the cause is B, then T has a normal distribution with parameters $\mu_B = 4$ hours but the same σ parameter.

   From the probability tables provided for the standard Normal distribution ($\mu = 0$ and $\sigma = 1$).

   (a) a failure from cause A takes longer than 3 hours to repair?

   (b) a machine which fails will take longer than 3 hours to repair?

   (c) there was a Type A failure, **given** that the repair time is longer than 3 hours.

   Hint: For (b) and (c), use the two probability theorems.

7. The *median* of a continuous random variable Y is that value y such that $F(y) = 1/2$. Find the median of Y when

   (a) Y has an *Exponential* distribution with parameter $\beta > 0$, that is

   $$f(y) = \frac{1}{\beta} e^{-y/\beta} \qquad y > 0$$

   and zero otherwise.

   (b) $\ln Y$ has a Normal distribution with parameters μ and σ^2.