MATH 323 - EXERCISES 1
NOT FOR ASSESSMENT.

EVENTS AND SAMPLE SPACES

1. Let A, B and C be three arbitrary events. Using only the operations of union, intersection and complement, write down expressions for the following events:

   (a) Only A occurs.
   (b) Both A and B, but not C occurs.
   (c) All three events occur.
   (d) At least one of A, B and C occurs.
   (e) At least two of A, B and C occur.
   (f) Precisely one of A, B and C occurs.
   (g) Precisely two of A, B and C occur.
   (h) None of A, B and C occurs.
   (i) Not more than two of A, B and C occur.

2. Let A, B and C be three arbitrary events. Which of the following relationships are true? Justify your answers.

   (a) $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$.
   (b) $(A \cup B) = (A \cap B') \cup B$.
   (c) $(A' \cap B) \cup (A \cap B') = (A \cup B) \cap (A \cap B)'$.
   (d) $(A \cup B)' \cap C = A' \cap B' \cap C'$.
   (e) $(A \cap B) \cap (B' \cap C) = \emptyset$.

3. A coin is tossed three times. Let A be the event that there are exactly two heads, B the event that there are more heads than tails, and C the event that the last toss is a tail. Using the operations of union, intersection and complement, find in terms of A, B and C expressions for the events:

   (a) There are more tails than heads.
   (b) There are three heads.
   (c) The first two tosses are heads.

4. Items from a production line are individually judged to be either satisfactory (S), or faulty (F). Inspection continues until two consecutive faulty items are encountered, or a total of four items have been checked, whichever occurs first.

   (a) Describe the sample space for this situation.
   (b) Write down the elementary outcomes in the event

   "the final item inspected was satisfactory".

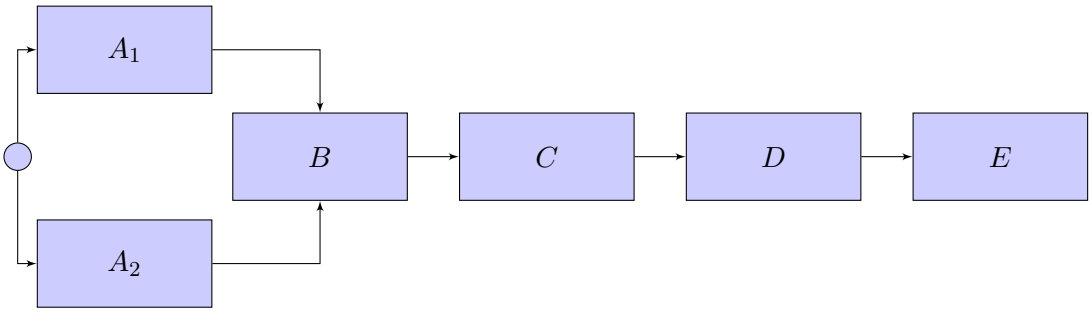
5. A box of n light bulbs contains r with broken filaments ($r < n$). Describe the sample space for the following situations: bulbs are tested, one by one, until

   (a) a defective is found;
   (b) all defectives are found.

6. A schematic diagram for a complex system, a space module, is depicted below;

MAIN ENGINE	—	SERVICE EXPULSION SYSTEM	—	COMMAND SERVICE MODULE	—	LUNAR EXPULSION MODULE (LEM)	—	LEM ENGINE
A		B		C		D		E

Thought of as a sequence of components, which either work or do not work, and given that there is a parallel standby main engine A_2 which takes over if the initially ignited main engine A_1 fails, we have the following representation:



(a) Identifying the event that a given unit functions with corresponding letter, give a representation of the event

"entire system functions".

(b) What does this representation become if B actually consists of three components, B_1, B_2 and B_3, and functions *only if at least two* of these function?