MATH 323 - ASSIGNMENT 3

Please submit your assignment by 11.59 pm on Friday 5th November by uploading a pdf to myCourses.

1. Suppose that the number of eggs produced by a particular species of fly in a given batch is a discrete random variable Y with pmf

$$p(y) = \begin{cases} c\dfrac{p^y}{y} & y \in \{1, 2, 3, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

   for parameter p, $0 < p < 1$, and some constant c.

   (a) By finding an appropriate value of constant c, show that this is a valid pmf. 2 MARKS

   (b) Find the probability $P(Y > 4)$ if $p = 0.2$. 2 MARKS

   (c) For this distribution, which is bigger, $\mathbb{E}[Y]$ or $\mathbb{E}[Y^2]$? Justify your answer. 4 MARKS

2. Suppose that two random variables Y_1 and Y_2 are *independent*, that is, for all values of y_1 and y_2,

$$P((Y_1 = y_1) \cap (Y_2 = y_2)) = P(Y_1 = y_1)P(Y_2 = y_2)$$

   that is, the events $(Y_1 = y_1)$ and $(Y_2 = y_2)$ are independent. Suppose also that Y_1 and Y_2 have the same Geometric distribution, that is $Y_1 \sim Geometric(p)$ and $Y_2 \sim Geometric(p)$. Define a third random variable Y as the sum of Y_1 and Y_2, that is, $Y = Y_1 + Y_2$.

   By considering the Theorem of Total Probability and the partitioning result

$$(Y = y) \equiv \bigcup_{t=1}^{y-1} ((Y_1 = t) \cap (Y_2 = y - t))$$

   for $y = 2, 3, \dots$, derive the pmf of Y. 6 MARKS

   Hint: apply Axiom III to the above partition, and use the pmfs for Y_1 and Y_2, and the assumed independence, to compute the right hand side..

3. Suppose that $Y \sim Binomial(n, p)$ (that is, discrete random variable Y has a Binomial distribution with parameters n and p, where n is a (strictly) positive integer, and $0 < p < 1$).

   (a) Verify using general properties of expectations that

$$\mathbb{E}[Y^3] = \mathbb{E}[Y(Y-1)(Y-2)] + 3\mathbb{E}[Y(Y-1)] + \mathbb{E}[Y]$$

   2 MARKS

   (b) Find an expression for $\mathbb{E}[Y^3]$ for $n \geq 3$. Justify your answer. 4 MARKS