MATH 323 - ASSIGNMENT 2

Please submit your assignment by 11.59 pm on Friday 8th October by uploading a pdf to myCourses Assignment 2.

Note: this assignment will be hand-graded, so any legible numerical answer format is acceptable.

1. Suppose that A and B are events in sample space S. Using the stated information, compute the requested conditional probability. Show your working. If the requested conditional probability cannot be computed due to lack of information or due to contradictory information, explain why. If the requested conditional probability is not defined, explain why.

   (a) $P(A) = 0.423$, $P(B) = 0.190$, $P(A \cap B) = 0.115$. Compute $P(B|A)$ 1 MARK

   (b) $P(A \cup B) = 0.157$, $P(A \cap B') = 0.659$, $P(A' \cap B) = 0.044$. Compute $P(A|B)$. 2 MARKS

   (c) $P(A' \cap B) = 0.420$, $P(A \cap B') = 0.580$. Compute $P(A|B)$. 2 MARKS

   (d) $P(A) = 0.050$, $P(B) = 0.891$, $P(A \cap B) = 0.046$. Compute $P(B'|A)$. 2 MARKS

   (e) $P(A) = 0.635$, $P(B) = 0.433$, $P(A \cap B) = 0.210$. Compute $P(A'|B')$. 2 MARKS

2. The operating lifetime (in hours) of a phone battery can be described using a continuous sample space $S = \mathbb{R}^+$ (ie the positive real numbers) by considering events of the type

   $$A_x = \text{“Lifetime of battery exceeds (ie is greater than or equal to) } x \text{ hours”}$$

   and by specifying probabilities of events of this type using the formula

   $$P(A_x) = \exp\{-(x/\lambda)^2\} \qquad x > 0$$

   where λ is a positive constant value. For the purpose of the question, $\lambda = 2000$.

   (a) Compute the probability that the lifetime exceeds 1000 hours. 1 MARK

   (b) Compute the probability that the lifetime exceeds 1000 hours if it is **known** that the lifetime exceeds 500 hours. 2 MARKS

   (c) If five identical phone batteries were used under identical operating conditions in five different phones, so that any events concerning the battery lifetimes could be considered mutually independent, compute that probability that **one or more** of the battery lifetimes did **not** exceed 500 hours. 2 MARKS

Please turn over for Question 3.

3. A bag contains five coins: two are standard (Head/Tail), but two are double-headed (Head/Head) and one is double-tailed (Tail/Tail). When any of the coins is tossed, each side is equally likely to land face-up.

   (a) A coin is selected from the bag and tossed, with all coins equally likely to be selected. Compute the probability that the side that is face-down is a Head. 2 MARKS

   (b) After the first toss, it is observed that the visible (face-up) side is a Head. Given this information, compute the probability that the side that is face-down is a Head. 2 MARKS

   (c) The same selected coin is then tossed again. Compute the probability that the side that is face-down after the second toss is a Head, given the information gained after the first toss.

   2 MARKS