MATH 323 - ASSIGNMENT 1

Please submit your assignment by 11.59 pm on Friday 24th September by entering the numerical answers into Quiz 1 in myCourses.



The following Venn diagram depicts the partition of sample space S created by two events A and B.

1. For each of parts (a) to (e), using the information given in the question, find the numerical value of probability requested. If there is insufficient information to find the requested probability, or the information in the question is invalid or contradictory, report the value NA.

(a) Find P(A) if

$$P(A \cap B') = 0.37$$
 $P(A' \cap B) = 0.09$ $P(A' \cap B') = 0.22$

Answer: P(A) = 0.69. We have by de Morgan that $P(A \cup B) = 1 - P(A' \cap B') = 0.78$, and as

$$P(A \cup B) = P(A \cap B') + P(A \cap B) + P(A' \cap B)$$

we must have $P(A \cap B) = 0.78 - 0.37 - 0.09 = 0.32$. Thus

 $P(A) = P(A \cap B') + P(A \cap B) = 0.37 + 0.32 = 0.69.$

2 MARKS

(b) Find $P(A \cap B)$ if

$$P(A) = 0.52$$
 $P(A \cap B') = 0.24$
Answer: $P(A \cap B) = 0.28$. By partitioning, $P(A) = P(A \cap B') + P(A \cap B)$, so
 $P(A \cap B) = P(A) - P(A \cap B') = 0.52 - 0.24 = 0.28.$

2 MARKS

(c) Find $P(A \cup B)$ if

P(A) = 0.80 P(B) = 0.25 $P(A' \cap B') = 0.04$ Answer: $P(A \cup B) = 0.96$. By de Morgan, $P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.04 = 0.96$. 2 MARKS (d) Find the probability of the event

 $(A \cap B') \cup (A' \cap B)$

 $P(A \cap B) = 0.20$ $P(A' \cap B') = 0.28$

Answer: $P((A \cap B') \cup (A' \cap B)) = 0.52$.

(e) Find $P(A \cap B)$ if

$$P(A) = 0.78$$
 $P(B) = 0.31$ $P(A \cup B) = 0.86.$

Answer: $P(A \cap B) = 0.23$. We have by the General Addition Rule

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.78 + 0.31 - 0.86 = 0.23.$$

2 MARKS

- 2. In 2018, a survey of 1000 high-school students was carried out to establish the amount of usage of three social media platforms. Each student was asked whether they were regular users of
 - A WhatsApp
 - B Instagram
 - C TikTok

The survey results are reported in the following table:

A	В	С	Count
Y	Y	Y	262
Y	Y	Ν	230
Y	Ν	Y	139
N	Y	Y	139
Y	Ν	Ν	96
N	Y	Ν	70
N	Ν	Y	55
N	Ν	Ν	9

For example, 230 participants reported being regular users of WhatsApp and Instagram but not TikTok.

One participant is selected for follow-up, with all participants being equally likely to be selected. Denoting by *A*, *B* and *C* the events that the selected participant reports using platform A, B and C respectively, compute

(a)	P(A) Answer: (262+230+139+96)/1000 = 0.727.	2 MARKS
(b)	$P(A \cup B \cup C)$ Answer: 1-9/1000 = 0.991.	2 MARKS
(c)	$P((A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C))$ Answer: (230+139+139)/1000 = 0.508.	2 MARKS

Report your answers to three decimal places.

2 MARKS

3. A game of dice is played by repeatedly rolling a fair die (where outcomes $\{1, 2, 3, 4, 5, 6\}$ are equally likely on each roll, and the result of any roll does not affect the result of any other roll) until the number 1 or the number 6 is rolled. What is the probability that the game ends on an odd-numbered of roll?

Answer: We have by equally likely outcomes that

$$P(E_i) = \frac{4^{i-1} \times 2}{6^i} = \left(\frac{2}{3}\right)^{i-1} \left(\frac{1}{3}\right).$$

as, by the multiplication principle,

- for the numerator, we must have a sequence of (i-1) rolls of $\{2, 3, 4, 5\}$, and then finally a roll of $\{1, 6\}$;
- for the denominator, we have a sequence of i rolls of the numbers $\{1, 2, 3, 4, 5, 6\}$.

Then using Axiom (III), we have

$$P(A) = \sum_{i=0}^{\infty} P(E_{2i+1}) = \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{2i} \left(\frac{1}{3}\right) = \frac{1}{3} \times \frac{1}{1-4/9} = \frac{1}{3} \times \frac{9}{5} = \frac{3}{5}.$$

summing the geometric progression..