MATH 323 - ASSIGNMENT 0
(NOT AN ASSIGNMENT)

1. If
$$P(A) = 0.67 \qquad P(B) = 0.45 \qquad P(A \cap B) = 0.21$$
find $P(A \cup B)$.

Answer: by the general addition rule
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.67 + 0.45 - 0.21 = 0.91$$
2 MARKS

2. If $P(A) = 0.7$ and $P(B) = 0.6$, what is the largest numerical value that $P(A \cap B)$ can take ? What is the smallest value that $P(A \cup B)$ can take ?

Answer: $P(A \cap B)$ can be as large as 0.60 – which occurs if B is a proper subset of A; in general we know that $A \cap B$ is a subset of A and B, and therefore
$$P(A \cap B) \leq \min\{P(A), P(B)\}$$
but we have no further information to make the bound even lower.

$P(A \cup B)$ can be as small as 0.70 – which occurs if B is a proper subset of A. By the general addition rule, we have that
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A)$$
if $P(B) = P(A \cap B)$.
2 MARKS

3. Compute $P(A)$ if
$$P(A \cap B) = 0.25 \quad P(A' \cap B) = 0.15 \quad P(A \cap B') = 0.50$$

Answer: we have by partitioning that
$$A = (A \cap B) \cup (A \cap B')$$
and by Axiom (III)
$$P(A) = P(A \cap B) + P(A \cap B') = 0.25 + 0.50 = 0.75.$$
2 MARKS

4. If
$$P(A) = 0.20 \qquad P(B) = 0.60 \qquad P(A \cap B) = 0.10$$
find $P(A' \cap B')$.

Answer: by de Morgan's Laws,
$$(A' \cap B') = (A \cup B)'$$
so therefore
$$P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = 1 - 0.20 - 0.60 + 0.10 = 0.30.$$
2 MARKS

5. If $P(A \cap B) = 0.80$, what is the largest value that the probability that "precisely one of A and B occurs" can take ?

   Answer: The event of interest is
   $$(A \cap B') \cup (A' \cap B).$$
   In the fundamental partition of S
   $$S = (A \cap B') \cup (A \cap B) \cup (A' \cap B) \cup (A' \cap B')$$
   that yields
   $$P(S) = P(A \cap B') + P(A \cap B) + P(A' \cap B) + P(A' \cap B') = 1$$
   we have by the partition that
   $$P((A \cap B') \cup (A' \cap B)) = P(A \cap B') + P(A' \cap B)$$
   which can be made as large as 0.20 by setting $P(A' \cap B') = 0$. This corresponds to having $A \cup B = S$.
   2 MARKS