Q1: A survey of grocery store customers concerning their purchasing habits for three brands of breakfast cereal labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	21
N	N	Y	20
N	Y	N	23
Y	N	N	26
N	Y	Y	21
Y	N	Y	22
Y	Y	N	19
Y	Y	Y	17
	Total		169

That is, 21 had purchased none of the brands, 21 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 84/169 = 0.4970414

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 22/169 = 0.1301775

(c) purchased at least two of the brands; 2 MARKS

Solution: 79/169 = 0.4674556

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A)$ if

$$P(A \cap B') = 0.368 \qquad P(A' \cap B) = 0.42 \qquad P(A' \cap B') = 0.099$$

Answer: $P(A) = 0.481$. 1 MARK

(e) Find $P(A \cup B)$ if

$$P(A) = 0.224 \qquad P(B) = 0.238 \qquad P(A' \cap B') = 0.66$$

Answer: $P(A \cup B) = 0.34$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.63 \qquad P(A \cap B') = 0.233$$

Answer: $P(A \cap B) = 0.397$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.418 \qquad P(A' \cap B') = 0.276$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.276 - 0.418 \\ &= 0.306 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 9$ items from $n = 10$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 3628800/1 = 3628800$ or $C_r^n = 3628800/(362880 \times 1) = 10$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 11$ are there that contain $r = 9$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{11}{9} = 55$$

2 MARKS

(c) How many ways are there of arranging the letters of the word BAMBOO into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 2, 2, with 6 letters in total so

$$180$$

2 MARKS

(d) How many distinct committees of sizes 2, 3, 4 and 5 can be formed from a collection of 14 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{14!}{2!\ 3!\ 4!\ 5!} = 2522520$$

2 MARKS

(e) There are $N = 10$ pieces of fruit in a bag, and $n = 6$ are selected without replacement, with all selections being equally likely. If the bag contains $R = 3$ apples and $N - R = 7$ oranges, what is the probability that there are $r = 1$ apples among the selected fruit ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{3}{1}\binom{7}{5}}{\binom{10}{6}} = 0.3$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.324$, $P(A \cap B') = 0.14$, $P(A' \cap B) = 0.102$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.324}{0.324 + 0.102} = 0.76$$

(b) $P(B) = 0.728$, $P(A' \cap B) = 0.326$, $P(A \cap B') = 0.144$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.728 - 0.326}{0.728} = 0.553$$

(c) A construction company buys its wood from three suppliers; Supplier 1 provides 16% of the wood, Supplier 2 provides 25% of the wood, and Supplier 3 provides 59% of the wood.

Of all of the wood shipments supplied

- 2% of shipments from Supplier 1 do not meet minimum quality standards;
- 15% of shipments from Supplier 2 do not meet minimum quality standards;
- 31% of shipments from Supplier 3 do not meet minimum quality standards;

(i) What is the probability that a given shipment does not meet minimum quality standards ? 2 MARKS

Solution:

$$P(F) = \sum_{i=1}^{3} P(F|A_i)P(A_i) = (0.16 \times 0.02) + (0.25 \times 0.15) + (0.59 \times 0.31) = 0.2236$$

(ii) A shipment is discovered not to meet minimum quality standards. Given this information, what is the probability that it was supplied by Supplier 1 ? 2 MARKS

Solution:

$$P(A_1|F) = \frac{P(F|A_1)P(A_1)}{P(F)} = \frac{0.16 \times 0.02}{0.2236} = 0.0143113$$

(iii) The construction company decides to stop using Supplier 3 due to low quality, but allows Supplier 1 and Supplier 2 to provide equal proportions of shipments subsequently. Several months later, under the new system, a shipment is discovered to fail the minimum quality standards. What is the probability that it was supplied by Supplier 2 ? 2 MARKS

Solution:

$$P(F) = P(F|A_1)\frac{1}{2} + P(F|A_2)\frac{1}{2} = 0.02 \times \frac{1}{2} + 0.15 \times \frac{1}{2} = 0.085$$

so therefore

$$P(A_2|F) = \frac{P(F|A_2)P(A_2)}{P(F)} = \frac{0.15 \times \frac{1}{2}}{0.085} = 0.8823529$$

Q1: A survey of cafe customers concerning their purchasing habits for three brands of coffee labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	23
N	N	Y	14
N	Y	N	16
Y	N	N	14
N	Y	Y	20
Y	N	Y	25
Y	Y	N	17
Y	Y	Y	20
	Total		149

That is, 23 had purchased none of the brands, 20 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 76/149 = 0.5100671

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 25/149 = 0.1677852

(c) purchased at least two of the brands; 2 MARKS

Solution: 82/149 = 0.5503356

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A \cup B)$ if

$$P(A) = 0.722 \qquad P(B) = 0.273 \qquad P(A' \cap B') = 0.178$$

Answer: $P(A \cup B) = 0.822$. 1 MARK

(e) Find $P(B)$ if

$$P(A \cap B') = 0.15 \qquad P(A' \cap B) = 0.404 \qquad P(A' \cap B') = 0.043$$

Answer: $P(B) = 0.807$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.519 \qquad P(A \cap B') = 0.182$$

Answer: $P(A \cap B) = 0.337$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.275 \qquad P(A' \cap B') = 0.335$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.335 - 0.275 \\ &= 0.39 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 8$ items from $n = 9$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 362880/1 = 362880$ or $C_r^n = 362880/(40320 \times 1) = 9$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 10$ are there that contain $r = 4$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{10}{4} = 210$$

2 MARKS

(c) How many ways are there of arranging the letters of the word CALICO into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 1, 1, 2, with 6 letters in total so

$$360$$

2 MARKS

(d) How many distinct committees of sizes 3, 4, 5 and 5 can be formed from a collection of 17 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{17!}{3!\ 4!\ 5!\ 5!} = 171531360$$

2 MARKS

(e) There are $N = 12$ pieces of fruit in a bag, and $n = 8$ are selected without replacement, with all selections being equally likely. If the bag contains $R = 3$ apples and $N - R = 9$ oranges, what is the probability that there are $r = 3$ apples among the selected fruit ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{3}{3}\binom{9}{5}}{\binom{12}{8}} = 0.2545455$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.156$, $P(A \cap B') = 0.149$, $P(A' \cap B) = 0.564$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.156}{0.156 + 0.564} = 0.217$$

(b) $P(B) = 0.39$, $P(A' \cap B) = 0.107$, $P(A \cap B') = 0.116$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.39 - 0.107}{0.39} = 0.726$$

(c) A construction company buys its wood from three suppliers; Supplier 1 provides 24% of the wood, Supplier 2 provides 38% of the wood, and Supplier 3 provides 38% of the wood.

Of all of the wood shipments supplied

- 7% of shipments from Supplier 1 do not meet minimum quality standards;
- 19% of shipments from Supplier 2 do not meet minimum quality standards;
- 27% of shipments from Supplier 3 do not meet minimum quality standards;

(i) What is the probability that a given shipment does not meet minimum quality standards ? 2 MARKS

Solution:

$$P(F) = \sum_{i=1}^{3} P(F|A_i)P(A_i) = (0.24 \times 0.07) + (0.38 \times 0.19) + (0.38 \times 0.27) = 0.1916$$

(ii) A shipment is discovered not to meet minimum quality standards. Given this information, what is the probability that it was supplied by Supplier 1 ? 2 MARKS

Solution:

$$P(A_1|F) = \frac{P(F|A_1)P(A_1)}{P(F)} = \frac{0.24 \times 0.07}{0.1916} = 0.0876827$$

(iii) The construction company decides to stop using Supplier 3, and allows Supplier 1 to supply twice as many contracts as Supplier 2. What is the probability that a given shipment does not meet minimum quality standards under the new system ? 2 MARKS

Solution:

$$P(F) = P(F|A_1)\frac{2}{3} + P(F|A_2)\frac{1}{3} = 0.07 \times \frac{2}{3} + 0.19 \times \frac{1}{3} = 0.11$$

Q1: A survey of office supply store customers concerning their purchasing habits for three brands of paper labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	33
N	N	Y	25
N	Y	N	25
Y	N	N	20
N	Y	Y	17
Y	N	Y	16
Y	Y	N	23
Y	Y	Y	23
	Total		182

That is, 33 had purchased none of the brands, 17 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 82/182 = 0.4505495

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 16/182 = 0.0879121

(c) purchased at least two of the brands; 2 MARKS

Solution: 79/182 = 0.4340659

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A \cup B)$ if

$$P(A) = 0.681 \qquad P(B) = 0.234 \qquad P(A' \cap B') = 0.152$$

Answer: $P(A \cup B) = 0.848$. 1 MARK

(e) Find $P(B)$ if

$$P(A \cap B') = 0.501 \qquad P(A' \cap B) = 0.153 \qquad P(A' \cap B') = 0.292$$

Answer: $P(B) = 0.207$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.581 \qquad P(A \cap B') = 0.188$$

Answer: $P(A \cap B) = 0.393$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.281 \qquad P(A' \cap B') = 0.319$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.319 - 0.281 \\ &= 0.4 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 7$ items from $n = 8$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 40320/1 = 40320$ or $C_r^n = 40320/(5040 \times 1) = 8$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 10$ are there that contain $r = 3$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{10}{3} = 120$$

2 MARKS

(c) How many ways are there of arranging the letters of the word TENET into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 2, 2, with 5 letters in total so

$$30$$

2 MARKS

(d) How many distinct committees of sizes 3, 3, 5 and 5 can be formed from a collection of 16 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{16!}{3!\ 3!\ 5!\ 5!} = 40360320$$

2 MARKS

(e) A committee of size $n = 6$ is to be formed from $R = 3$ students from Faculty I and $N - R = 9$ students from Faculty II, where $N = 12$. What is the probability that there are $r = 2$ Faculty I students on the committee, if the committee members are selected so that all selections are equally likely ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{3}{2}\binom{9}{4}}{\binom{12}{6}} = 0.4090909$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.024$, $P(A \cap B') = 0.346$, $P(A' \cap B) = 0.043$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.024}{0.024 + 0.043} = 0.352$$

(b) $P(B) = 0.374$, $P(A' \cap B) = 0.273$, $P(A \cap B') = 0.237$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.374 - 0.273}{0.374} = 0.27$$

(c) Suppose that in any year, a person can receive up to two vaccine shots to prevent flu.

- If the person gets no flu shots, then they will get the flu with probability 0.22;
- if they get the first shot only, they will get the flu with probability 0.07;
- if they get the first and second shots, they will get the flu with probability 0.06.

Suppose that it is known that 21% of people do not get a flu shot, 25% of people get the first flu shot only, and 54% of people get the first and second flu shots.

(i) What is the probability that a person in the population will get the flu ? 2 MARKS

Solution:

$$P(F) = \sum_{i=1}^{3} P(F|A_i)P(A_i) = (0.21 \times 0.22) + (0.25 \times 0.07) + (0.54 \times 0.06) = 0.0961$$

(ii) Given that a person is observed to get the flu, what is the probability that the person got the first and second flu shots ? 2 MARKS

Solution:

$$P(A_3|F) = \frac{P(F|A_3)P(A_3)}{P(F)} = \frac{0.54 \times 0.06}{0.0961} = 0.3371488$$

(iii) Given that a person is observed to get the flu, **and** it is also known that they received at most one flu shot, what is the probability that they did not receive any flu shot ? 2 MARKS

Solution: by disjointness

$$P(A_1|C \cap (A_1 \cup A_2)) = \frac{P(A_1 \cap C)}{P(A_1 \cap C) + P(A_2 \cap C)} = \frac{P(C|A_1)P(A_1)}{P(C|A_1)P(A_1) + P(C|A_2)P(A_2)} = \frac{0.21 \times 0.22}{0.0637} = 0.7252747$$

Q1: A survey of office supply store customers concerning their purchasing habits for three brands of paper labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	18
N	N	Y	20
N	Y	N	24
Y	N	N	20
N	Y	Y	28
Y	N	Y	20
Y	Y	N	28
Y	Y	Y	34
	Total		192

That is, 18 had purchased none of the brands, 28 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 102/192 = 0.53125

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 20/192 = 0.1041667

(c) purchased at least two of the brands; 2 MARKS

Solution: 110/192 = 0.5729167

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A \cup B)$ if

$$P(A) = 0.671 \qquad P(B) = 0.511 \qquad P(A' \cap B') = 0.131$$

Answer: $P(A \cup B) = 0.869$. 1 MARK

(e) Find $P(B)$ if

$$P(A \cap B') = 0.158 \qquad P(A' \cap B) = 0.348 \qquad P(A' \cap B') = 0.242$$

Answer: $P(B) = 0.6$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.602 \qquad P(A \cap B') = 0.475$$

Answer: $P(A \cap B) = 0.126$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.348 \qquad P(A' \cap B') = 0.178$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.178 - 0.348 \\ &= 0.474 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 7$ items from $n = 9$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 362880/2 = 181440$ or $C_r^n = 362880/(5040 \times 2) = 36$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 8$ are there that contain $r = 3$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{8}{3} = 56$$

2 MARKS

(c) How many ways are there of arranging the letters of the word TENANT into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 2, 2, with 6 letters in total so

$$180$$

2 MARKS

(d) How many distinct committees of sizes 4, 4, 4 and 5 can be formed from a collection of 17 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{17!}{4!\ 4!\ 4!\ 5!} = 214414200$$

2 MARKS

(e) There are $N = 13$ pieces of fruit in a bag, and $n = 10$ are selected without replacement, with all selections being equally likely. If the bag contains $R = 5$ apples and $N - R = 8$ oranges, what is the probability that there are $r = 5$ apples among the selected fruit ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{5}{5}\binom{8}{5}}{\binom{13}{10}} = 0.1958042$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.017$, $P(A \cap B') = 0.673$, $P(A' \cap B) = 0.015$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.017}{0.017 + 0.015} = 0.521$$

(b) $P(B) = 0.378$, $P(A' \cap B) = 0.133$, $P(A \cap B') = 0.11$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.378 - 0.133}{0.378} = 0.649$$

(c) Four age groups (50–59 years, 60–69 years, 70–79 years, and 80 years and over) are under study to assess the proportion of individuals suffering from chronic obstructive pulmonary disorder (COPD) in the population of adults aged 50 years and over in a particular region. In increasing age order, the four age groups

- make up 12%, 29%, 21% and 38% of the study population respectively;
- have associated (conditional) probabilities of developing COPD 0.05, 0.07, 0.41 and 0.5 respectively.

(i) What is the probability that a person aged 50 years or over develops COPD ? 2 MARKS

Solution:

$$P(C) = \sum_{i=1}^{4} P(C|A_i)P(A_i) = (0.12 \times 0.05) + (0.29 \times 0.07) + (0.21 \times 0.41) + (0.38 \times 0.5) = 0.3024$$

(ii) A health record is discovered recording the fact that a person suffered from COPD, but the age of the person is omitted from the record. Given the COPD diagnosis, what is the probability that the person is aged 80 years or over ? 2 MARKS

Solution:

$$P(A_4|C) = \frac{P(C|A_4)P(A_4)}{P(C)} = \frac{0.38 \times 0.5}{0.3024} = 0.6283069$$

(iii) Five people aged 50 years and over are selected from the population. If the development of COPD for each of the five people can be regarded as a collection of mutually independent events, what is the probability that **at most one** of the five develops COPD ? 2 MARKS

Solution: the probability that one or zero develop COPD is

$$5(1 - P(C))^4 P(C) + (1 - P(C))^5 = 0.5232865$$

Q1: A survey of office supply store customers concerning their purchasing habits for three brands of paper labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	30
N	N	Y	27
N	Y	N	30
Y	N	N	31
N	Y	Y	18
Y	N	Y	21
Y	Y	N	26
Y	Y	Y	22
	Total		205

That is, 30 had purchased none of the brands, 18 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 100/205 = 0.4878049

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 21/205 = 0.102439

(c) purchased at least two of the brands; 2 MARKS

Solution: 87/205 = 0.4243902

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A)$ if

$$P(A \cap B') = 0.31 \qquad P(A' \cap B) = 0.41 \qquad P(A' \cap B') = 0.212$$

Answer: $P(A) = 0.378$. 1 MARK

(e) Find $P(A \cup B)$ if

$$P(A) = 0.412 \qquad P(B) = 0.52 \qquad P(A' \cap B') = 0.356$$

Answer: $P(A \cup B) = 0.644$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.389 \qquad P(A \cap B') = 0.276$$

Answer: $P(A \cap B) = 0.114$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.222 \qquad P(A' \cap B') = 0.106$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.106 - 0.222 \\ &= 0.672 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 3$ items from $n = 8$ without replacement ?
Answer: either $P_r^n = n!/(n-r)! = 40320/120 = 336$ or $C_r^n = 40320/(6 \times 120) = 56$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 12$ are there that contain $r = 2$ ones ?
Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{12}{2} = 66$$

2 MARKS

(c) How many ways are there of arranging the letters of the word BILLET into distinct letter patterns ?
Answer: Multinomial coefficient: letter counts are 1, 1, 1, 1, 2, with 6 letters in total so

$$360$$

2 MARKS

(d) How many distinct committees of sizes 2, 4, 5 and 5 can be formed from a collection of 16 people ?
Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{16!}{2!\ 4!\ 5!\ 5!} = 30270240$$

2 MARKS

(e) There are $N = 14$ fish in a pond: $R = 4$ of them are of one species (labelled I), the remaining $N - R = 10$ are of another species (labelled II). A catch of $n = 10$ fish is made (that is, n fish are extracted in one catch). What is the probability that there are $r = 1$ species I fish in the catch ?
Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{4}{1}\binom{10}{9}}{\binom{14}{10}} = 0.03996$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.12$, $P(A \cap B') = 0.154$, $P(A' \cap B) = 0.424$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.12}{0.12 + 0.424} = 0.22$$

(b) $P(B) = 0.266$, $P(A' \cap B) = 0.147$, $P(A \cap B') = 0.345$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.266 - 0.147}{0.266} = 0.447$$

(c) Suppose that in any year, a person can receive up to two vaccine shots to prevent flu.

- If the person gets no flu shots, then they will get the flu with probability 0.13;
- if they get the first shot only, they will get the flu with probability 0.07;
- if they get the first and second shots, they will get the flu with probability 0.05.

Suppose that it is known that 24% of people do not get a flu shot, 60% of people get the first flu shot only, and 16% of people get the first and second flu shots.

(i) What is the probability that a person in the population will get the flu ? 2 MARKS

Solution:

$$P(F) = \sum_{i=1}^{3} P(F|A_i)P(A_i) = (0.24 \times 0.13) + (0.6 \times 0.07) + (0.16 \times 0.05) = 0.0812$$

(ii) Given that a person is observed to get the flu, what is the probability that the person got the first and second flu shots ? 2 MARKS

Solution:

$$P(A_3|F) = \frac{P(F|A_3)P(A_3)}{P(F)} = \frac{0.16 \times 0.05}{0.0812} = 0.0985222$$

(iii) Given that a person is observed to get the flu, **and** it is also known that they received at most one flu shot, what is the probability that they did not receive any flu shot ? 2 MARKS

Solution: by disjointness

$$P(A_1|C \cap (A_1 \cup A_2)) = \frac{P(A_1 \cap C)}{P(A_1 \cap C) + P(A_2 \cap C)} = \frac{P(C|A_1)P(A_1)}{P(C|A_1)P(A_1) + P(C|A_2)P(A_2)} = \frac{0.24 \times 0.13}{0.0732} = 0.4262295$$

Q1: A survey of office supply store customers concerning their purchasing habits for three brands of paper labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	25
N	N	Y	24
N	Y	N	20
Y	N	N	14
N	Y	Y	27
Y	N	Y	25
Y	Y	N	25
Y	Y	Y	34
	Total		194

That is, 25 had purchased none of the brands, 27 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 98/194 = 0.5051546

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 25/194 = 0.128866

(c) purchased at least two of the brands; 2 MARKS

Solution: 111/194 = 0.5721649

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A \cup B)$ if

$$P(A) = 0.223 \qquad P(B) = 0.506 \qquad P(A' \cap B') = 0.436$$

Answer: $P(A \cup B) = 0.564$. 1 MARK

(e) Find $P(B)$ if

$$P(A \cap B') = 0.131 \qquad P(A' \cap B) = 0.112 \qquad P(A' \cap B') = 0.266$$

Answer: $P(B) = 0.603$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.688 \qquad P(A \cap B') = 0.293$$

Answer: $P(A \cap B) = 0.395$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.071 \qquad P(A' \cap B') = 0.317$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.317 - 0.071 \\ &= 0.612 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 6$ items from $n = 9$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 362880/6 = 60480$ or $C_r^n = 362880/(720 \times 6) = 84$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 8$ are there that contain $r = 4$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{8}{4} = 70$$

2 MARKS

(c) How many ways are there of arranging the letters of the word POTATO into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 2, 2, with 6 letters in total so

$$180$$

2 MARKS

(d) How many distinct committees of sizes 2, 3, 4 and 4 can be formed from a collection of 13 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{13!}{2!\ 3!\ 4!\ 4!} = 900900$$

2 MARKS

(e) A committee of size $n = 12$ is to be formed from $R = 6$ students from Faculty I and $N - R = 9$ students from Faculty II, where $N = 15$. What is the probability that there are $r = 2$ Faculty I students on the committee, if the committee members are selected so that all selections are equally likely ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{6}{2}\binom{9}{10}}{\binom{15}{12}} = 0$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.543$, $P(A \cap B') = 0.057$, $P(A' \cap B) = 0.073$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.543}{0.543 + 0.073} = 0.882$$

(b) $P(B) = 0.253$, $P(A' \cap B) = 0.201$, $P(A \cap B') = 0.587$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.253 - 0.201}{0.253} = 0.205$$

(c) Four age groups (50–59 years, 60–69 years, 70–79 years, and 80 years and over) are under study to assess the proportion of individuals suffering from chronic obstructive pulmonary disorder (COPD) in the population of adults aged 50 years and over in a particular region. In increasing age order, the four age groups

- make up 41%, 41%, 7% and 11% of the study population respectively;
- have associated (conditional) probabilities of developing COPD 0.05, 0.18, 0.25 and 0.33 respectively.

(i) What is the probability that a person aged 50 years or over develops COPD ? 2 MARKS

Solution:

$$P(C) = \sum_{i=1}^{4} P(C|A_i)P(A_i) = (0.41 \times 0.05) + (0.41 \times 0.18) + (0.07 \times 0.25) + (0.11 \times 0.33) = 0.1481$$

(ii) A health record is discovered recording the fact that a person suffered from COPD, but the age of the person is omitted from the record. Given the COPD diagnosis, what is the probability that the person is aged 80 years or over ? 2 MARKS

Solution:

$$P(A_4|C) = \frac{P(C|A_4)P(A_4)}{P(C)} = \frac{0.11 \times 0.33}{0.1481} = 0.2451047$$

(iii) Five people aged 50 years and over are selected from the population. If the development of COPD for each of the five people can be regarded as a collection of mutually independent events, what is the probability that **at most one** of the five develops COPD ? 2 MARKS

Solution: the probability that one or zero develop COPD is

$$5(1 - P(C))^4 P(C) + (1 - P(C))^5 = 0.8387$$

Q1: A survey of grocery store customers concerning their purchasing habits for three brands of breakfast cereal labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	28
N	N	Y	19
N	Y	N	27
Y	N	N	22
N	Y	Y	24
Y	N	Y	28
Y	Y	N	29
Y	Y	Y	32
Total			209

That is, 28 had purchased none of the brands, 24 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 111/209 = 0.5311005

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 28/209 = 0.1339713

(c) purchased at least two of the brands; 2 MARKS

Solution: 113/209 = 0.5406699

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A)$ if

$$P(A \cap B') = 0.129 \qquad P(A' \cap B) = 0.424 \qquad P(A' \cap B') = 0.098$$

Answer: $P(A) = 0.478$. 1 MARK

(e) Find $P(A \cup B)$ if

$$P(A) = 0.505 \qquad P(B) = 0.553 \qquad P(A' \cap B') = 0.275$$

Answer: $P(A \cup B) = 0.725$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.693 \qquad P(A \cap B') = 0.317$$

Answer: $P(A \cap B) = 0.377$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.144 \qquad P(A' \cap B') = 0.303$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.303 - 0.144 \\ &= 0.553 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 3$ items from $n = 8$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 40320/120 = 336$ or $C_r^n = 40320/(6 \times 120) = 56$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 10$ are there that contain $r = 3$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{10}{3} = 120$$

2 MARKS

(c) How many ways are there of arranging the letters of the word BANANAS into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 2, 3, with 7 letters in total so

$$420$$

2 MARKS

(d) How many distinct committees of sizes 2, 2, 5 and 5 can be formed from a collection of 14 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{14!}{2!\ 2!\ 5!\ 5!} = 1513512$$

2 MARKS

(e) There are $N = 13$ pieces of fruit in a bag, and $n = 10$ are selected without replacement, with all selections being equally likely. If the bag contains $R = 2$ apples and $N - R = 11$ oranges, what is the probability that there are $r = 2$ apples among the selected fruit ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{2}{2}\binom{11}{8}}{\binom{13}{10}} = 0.5769231$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.193$, $P(A \cap B') = 0.099$, $P(A' \cap B) = 0.324$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.193}{0.193 + 0.324} = 0.373$$

(b) $P(B) = 0.347$, $P(A' \cap B) = 0.109$, $P(A \cap B') = 0.29$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.347 - 0.109}{0.347} = 0.685$$

(c) Four age groups (50–59 years, 60–69 years, 70–79 years, and 80 years and over) are under study to assess the proportion of individuals suffering from chronic obstructive pulmonary disorder (COPD) in the population of adults aged 50 years and over in a particular region. In increasing age order, the four age groups

- make up 14%, 21%, 25% and 40% of the study population respectively;
- have associated (conditional) probabilities of developing COPD 0.06, 0.06, 0.21 and 0.22 respectively.

(i) What is the probability that a person aged 50 years or over develops COPD ? 2 MARKS

Solution:

$$P(C) = \sum_{i=1}^{4} P(C|A_i)P(A_i) = (0.14 \times 0.06) + (0.21 \times 0.06) + (0.25 \times 0.21) + (0.4 \times 0.22) = 0.1615$$

(ii) A health record is discovered recording the fact that a person suffered from COPD, but the age of the person is omitted from the record. Given the COPD diagnosis, what is the probability that the person is aged 80 years or over ? 2 MARKS

Solution:

$$P(A_4|C) = \frac{P(C|A_4)P(A_4)}{P(C)} = \frac{0.4 \times 0.22}{0.1615} = 0.5448916$$

(iii) Five people aged 50 years and over are selected from the population. If the development of COPD for each of the five people can be regarded as a collection of mutually independent events, what is the probability that **at most one** of the five develops COPD ? 2 MARKS

Solution: the probability that one or zero develop COPD is

$$5(1 - P(C))^4 P(C) + (1 - P(C))^5 = 0.8136584$$

Q1: A survey of grocery store customers concerning their purchasing habits for three brands of breakfast cereal labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	27
N	N	Y	30
N	Y	N	30
Y	N	N	27
N	Y	Y	21
Y	N	Y	18
Y	Y	N	26
Y	Y	Y	28
	Total		207

That is, 27 had purchased none of the brands, 21 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 99/207 = 0.4782609

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 18/207 = 0.0869565

(c) purchased at least two of the brands; 2 MARKS

Solution: 93/207 = 0.4492754

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A)$ if

$$P(A \cap B') = 0.462 \qquad P(A' \cap B) = 0.117 \qquad P(A' \cap B') = 0.191$$

Answer: $P(A) = 0.692$. 1 MARK

(e) Find $P(A \cup B)$ if

$$P(A) = 0.383 \qquad P(B) = 0.306 \qquad P(A' \cap B') = 0.381$$

Answer: $P(A \cup B) = 0.619$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.733 \qquad P(A \cap B') = 0.424$$

Answer: $P(A \cap B) = 0.308$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.107 \qquad P(A' \cap B') = 0.371$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.371 - 0.107 \\ &= 0.522 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 3$ items from $n = 8$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 40320/120 = 336$ or $C_r^n = 40320/(6 \times 120) = 56$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 11$ are there that contain $r = 10$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{11}{10} = 11$$

2 MARKS

(c) How many ways are there of arranging the letters of the word COLLEGE into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 2, 2, with 6 letters in total so

$$180$$

2 MARKS

(d) How many distinct committees of sizes 3, 3, 4 and 5 can be formed from a collection of 15 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{15!}{3!\ 3!\ 4!\ 5!} = 12612600$$

2 MARKS

(e) A committee of size $n = 7$ is to be formed from $R = 5$ students from Faculty I and $N - R = 6$ students from Faculty II, where $N = 11$. What is the probability that there are $r = 5$ Faculty I students on the committee, if the committee members are selected so that all selections are equally likely ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{5}{5}\binom{6}{2}}{\binom{11}{7}} = 0.0454545$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.156$, $P(A \cap B') = 0.65$, $P(A' \cap B) = 0.052$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.156}{0.156 + 0.052} = 0.75$$

(b) $P(B) = 0.499$, $P(A' \cap B) = 0.368$, $P(A \cap B') = 0.339$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.499 - 0.368}{0.499} = 0.263$$

(c) A construction company buys its wood from three suppliers; Supplier 1 provides 8% of the wood, Supplier 2 provides 49% of the wood, and Supplier 3 provides 43% of the wood.

Of all of the wood shipments supplied

- 10% of shipments from Supplier 1 do not meet minimum quality standards;
- 20% of shipments from Supplier 2 do not meet minimum quality standards;
- 26% of shipments from Supplier 3 do not meet minimum quality standards;

(i) What is the probability that a given shipment does not meet minimum quality standards ? 2 MARKS

Solution:

$$P(F) = \sum_{i=1}^{3} P(F|A_i)P(A_i) = (0.08 \times 0.1) + (0.49 \times 0.2) + (0.43 \times 0.26) = 0.2178$$

(ii) A shipment is discovered not to meet minimum quality standards. Given this information, what is the probability that it was supplied by Supplier 1 ? 2 MARKS

Solution:

$$P(A_1|F) = \frac{P(F|A_1)P(A_1)}{P(F)} = \frac{0.08 \times 0.1}{0.2178} = 0.0367309$$

(iii) The construction company decides to stop using Supplier 3, and allows Supplier 1 to supply twice as many contracts as Supplier 2. What is the probability that a given shipment does not meet minimum quality standards under the new system ? 2 MARKS

Solution:

$$P(F) = P(F|A_1)\frac{2}{3} + P(F|A_2)\frac{1}{3} = 0.1 \times \frac{2}{3} + 0.2 \times \frac{1}{3} = 0.1333333$$

Q1: A survey of cafe customers concerning their purchasing habits for three brands of coffee labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	29
N	N	Y	18
N	Y	N	33
Y	N	N	39
N	Y	Y	35
Y	N	Y	26
Y	Y	N	31
Y	Y	Y	27
	Total		238

That is, 29 had purchased none of the brands, 35 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 123/238 = 0.5168067

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 26/238 = 0.1092437

(c) purchased at least two of the brands; 2 MARKS

Solution: 119/238 = 0.5

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A \cup B)$ if

$$P(A) = 0.565 \qquad P(B) = 0.834 \qquad P(A' \cap B') = 0.094$$

Answer: $P(A \cup B) = 0.906$. 1 MARK

(e) Find $P(B)$ if

$$P(A \cap B') = 0.075 \qquad P(A' \cap B) = 0.432 \qquad P(A' \cap B') = 0.058$$

Answer: $P(B) = 0.867$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.69 \qquad P(A \cap B') = 0.393$$

Answer: $P(A \cap B) = 0.297$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.389 \qquad P(A' \cap B') = 0.068$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$\begin{aligned} P((A \cap B') \cup (A' \cap B)) &= P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B) \\ &= 1 - 0.068 - 0.389 \\ &= 0.543 \end{aligned}$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 7$ items from $n = 10$ without replacement ?
Answer: either $P_r^n = n!/(n-r)! = 3628800/6 = 604800$ or $C_r^n = 3628800/(5040 \times 6) = 120$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 8$ are there that contain $r = 6$ ones ?
Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{8}{6} = 28$$

2 MARKS

(c) How many ways are there of arranging the letters of the word LITTLE into distinct letter patterns ?
Answer: Multinomial coefficient: letter counts are 1, 1, 2, 2, with 6 letters in total so

$$180$$

2 MARKS

(d) How many distinct committees of sizes 2, 3, 4 and 5 can be formed from a collection of 14 people ?
Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{14!}{2!\ 3!\ 4!\ 5!} = 2522520$$

2 MARKS

(e) There are $N = 15$ fish in a pond: $R = 8$ of them are of one species (labelled I), the remaining $N - R = 7$ are of another species (labelled II). A catch of $n = 10$ fish is made (that is, n fish are extracted in one catch). What is the probability that there are $r = 4$ species I fish in the catch ?
Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{8}{4}\binom{7}{6}}{\binom{15}{10}} = 0.1631702$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.407$, $P(A \cap B') = 0.156$, $P(A' \cap B) = 0.272$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.407}{0.407 + 0.272} = 0.6$$

(b) $P(B) = 0.268$, $P(A' \cap B) = 0.089$, $P(A \cap B') = 0.353$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.268 - 0.089}{0.268} = 0.667$$

(c) A construction company buys its wood from three suppliers; Supplier 1 provides 22% of the wood, Supplier 2 provides 58% of the wood, and Supplier 3 provides 20% of the wood.

Of all of the wood shipments supplied

- 4% of shipments from Supplier 1 do not meet minimum quality standards;
- 23% of shipments from Supplier 2 do not meet minimum quality standards;
- 32% of shipments from Supplier 3 do not meet minimum quality standards;

(i) What is the probability that a given shipment does not meet minimum quality standards ? 2 MARKS

Solution:

$$P(F) = \sum_{i=1}^{3} P(F|A_i)P(A_i) = (0.22 \times 0.04) + (0.58 \times 0.23) + (0.2 \times 0.32) = 0.2062$$

(ii) A shipment is discovered not to meet minimum quality standards. Given this information, what is the probability that it was supplied by Supplier 1 ? 2 MARKS

Solution:

$$P(A_1|F) = \frac{P(F|A_1)P(A_1)}{P(F)} = \frac{0.22 \times 0.04}{0.2062} = 0.042677$$

(iii) The construction company decides to stop using Supplier 3, and allows Supplier 1 to supply twice as many contracts as Supplier 2. What is the probability that a given shipment does not meet minimum quality standards under the new system ? 2 MARKS

Solution:

$$P(F) = P(F|A_1)\frac{2}{3} + P(F|A_2)\frac{1}{3} = 0.04 \times \frac{2}{3} + 0.23 \times \frac{1}{3} = 0.1033333$$

Q1: A survey of office supply store customers concerning their purchasing habits for three brands of paper labelled B_1, B_2 and B_3 was carried out. Each customer was asked if they had purchased each of the brands within the last two years, and the total counts of response patterns were recorded.

The results are displayed in the following table: Y indicates that the brand had been purchased, N indicates that the brand had not been purchased:

B_1	B_2	B_3	Count
N	N	N	28
N	N	Y	41
N	Y	N	32
Y	N	N	20
N	Y	Y	32
Y	N	Y	33
Y	Y	N	35
Y	Y	Y	29
	Total		250

That is, 28 had purchased none of the brands, 32 had purchased B_2 and B_3, but not B_1 etc.

A customer is selected from the survey participants, with all customers equally likely to be selected. Compute the probability that the customer had, in the last two years,

(a) purchased brand B_1 1 MARK

Solution: 117/250 = 0.468

(b) purchased brand B_1 and brand B_3, but not brand B_2; 1 MARK

Solution: 33/250 = 0.132

(c) purchased at least two of the brands; 2 MARKS

Solution: 129/250 = 0.516

In the following question compute the required probabilities. If the required probability cannot be computed due to lack of information or due to contradictory information, explain briefly why.

(d) Find $P(A \cup B)$ if

$$P(A) = 0.559 \qquad P(B) = 0.296 \qquad P(A' \cap B') = 0.26$$

Answer: $P(A \cup B) = 0.74$. 1 MARK

(e) Find $P(B)$ if

$$P(A \cap B') = 0.257 \qquad P(A' \cap B) = 0.11 \qquad P(A' \cap B') = 0.197$$

Answer: $P(B) = 0.546$. 1 MARK

(f) Find $P(A \cap B)$ if

$$P(A) = 0.55 \qquad P(A \cap B') = 0.166$$

Answer: $P(A \cap B) = 0.383$. 2 MARKS

(g) Find the probability of the event $(A \cap B') \cup (A' \cap B)$ if

$$P(A \cap B) = 0.142 \qquad P(A' \cap B') = 0.156$$

2 MARKS

Answer: as $(A \cap B') \cup (A' \cap B) = (A \cup B) \cap (A \cap B)'$ and $P(A' \cap B') = 1 - P(A \cup B)$ by de Morgan, we can write the required probability using the set difference

$$P((A \cap B') \cup (A' \cap B)) = P(A \cup B) - P(A \cap B) = 1 - P(A' \cap B') - P(A \cap B)$$

$$= 1 - 0.156 - 0.142$$

$$= 0.703$$

Q2: Give numerical solutions to the following questions, and a brief justification

(a) How many ways are there of selecting $r = 5$ items from $n = 8$ without replacement ?

Answer: either $P_r^n = n!/(n-r)! = 40320/6 = 6720$ or $C_r^n = 40320/(120 \times 6) = 56$ 2 MARKS

(b) How many different binary sequences (that is, sequences of zeros and ones) of length $n = 9$ are there that contain $r = 6$ ones ?

Answer: Binomial coefficient:

$$\binom{n}{r} = \binom{9}{6} = 84$$

2 MARKS

(c) How many ways are there of arranging the letters of the word MASSES into distinct letter patterns ?

Answer: Multinomial coefficient: letter counts are 1, 1, 1, 3, with 6 letters in total so

$$120$$

2 MARKS

(d) How many distinct committees of sizes 3, 3, 3 and 5 can be formed from a collection of 14 people ?

Answer: Multinomial coefficient

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{14!}{3!\ 3!\ 3!\ 5!} = 3363360$$

2 MARKS

(e) A committee of size $n = 7$ is to be formed from $R = 5$ students from Faculty I and $N - R = 5$ students from Faculty II, where $N = 10$. What is the probability that there are $r = 3$ Faculty I students on the committee, if the committee members are selected so that all selections are equally likely ?

Answer: Hypergeometric probability

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} = \frac{\binom{5}{3}\binom{5}{4}}{\binom{10}{7}} = 0.4166667$$

2 MARKS

Q3: In this question compute the required probabilities and conditional probabilities, and show your working. If the required probability cannot be computed due to lack of information or due to contradictory information, explain why. If the required probability is not defined, explain why.

(a) $P(A \cap B) = 0.339$, $P(A \cap B') = 0.201$, $P(A' \cap B) = 0.266$. Compute $P(A|B)$. 2 MARKS

Solution: Here we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)} = \frac{0.339}{0.339 + 0.266} = 0.561$$

(b) $P(B) = 0.885$, $P(A' \cap B) = 0.54$, $P(A \cap B') = 0.063$. Compute $P(A|B)$. 2 MARKS

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) - P(A' \cap B)}{P(B)} = \frac{0.885 - 0.54}{0.885} = 0.39$$

(c) Four age groups (50–59 years, 60–69 years, 70–79 years, and 80 years and over) are under study to assess the proportion of individuals suffering from chronic obstructive pulmonary disorder (COPD) in the population of adults aged 50 years and over in a particular region. In increasing age order, the four age groups

- make up 21%, 62%, 6% and 11% of the study population respectively;
- have associated (conditional) probabilities of developing COPD 0.02, 0.07, 0.08 and 0.17 respectively.

(i) What is the probability that a person aged 50 years or over develops COPD ? 2 MARKS

Solution:

$$P(C) = \sum_{i=1}^{4} P(C|A_i)P(A_i) = (0.21 \times 0.02) + (0.62 \times 0.07) + (0.06 \times 0.08) + (0.11 \times 0.17) = 0.0711$$

(ii) A health record is discovered recording the fact that a person suffered from COPD, but the age of the person is omitted from the record. Given the COPD diagnosis, what is the probability that the person is aged 80 years or over ? 2 MARKS

Solution:

$$P(A_4|C) = \frac{P(C|A_4)P(A_4)}{P(C)} = \frac{0.11 \times 0.17}{0.0711} = 0.2630098$$

(iii) Five people aged 50 years and over are selected from the population. If the development of COPD for each of the five people can be regarded as a collection of mutually independent events, what is the probability that **at most one** of the five develops COPD ? 2 MARKS

Solution: the probability that one or zero develop COPD is

$$5(1 - P(C))^4 P(C) + (1 - P(C))^5 = 0.9562603$$