

MATH 323 - AN APPLICATION OF THE PROBABILITY THEOREMS

THE MONTY HALL GAME SHOW PROBLEM

Problem: In a TV Game show, a contestant selects one of three doors; behind one of the doors there is a prize, and behind the other two there are no prizes. After the contestant selects a door, the game-show host opens one of the remaining doors, and reveals that there is no prize behind it. The host then asks the contestant whether they want to SWITCH their choice to the other unopened door, or STICK to their original choice.

Is it probabilistically advantageous for the contestant to SWITCH doors, or is the probability of winning the prize the same whether they STICK or SWITCH ? (Assume that the host selects a door to open, from those available, with equal probability).

Solution of this problem requires a simple application of conditional probability and Bayes Theorem, and illustrates that the correct answer is occasionally counter-intuitive.

Solution: Without loss of generality, let events A, B, C correspond to the prize being behind the selected, opened, and remaining door respectively, and let H_B denote the event that the host opens door B . We want to compare $P(A|H_B)$ (STICK) with $P(C|H_B)$ (SWITCH).

We have

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

and also

$$P(H_B|A) = \frac{1}{2} \quad P(H_B|B) = 0 \quad P(H_B|C) = 1.$$

Then the extended version of Bayes theorem gives

$$\begin{aligned} P(A|H_B) &= \frac{P(H_B|A)P(A)}{P(H_B)} = \frac{P(H_B|A)P(A)}{P(H_B|A)P(A) + P(H_B|B)P(B) + P(H_B|C)P(C)} \\ &= \frac{\left(\frac{1}{2} \times \frac{1}{3}\right)}{\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(0 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{3}\right)} = \frac{1}{3} \end{aligned}$$

so $P(A|H_B) = 1/3$, and similarly $P(C|H_B) = 2/3$. so it is advantageous to SWITCH.