

MATH 323 - ASSIGNMENT 1

Please Hand in Assignment by 5pm on Monday 12th May.

- 1 Three newspapers, denoted A, B and C are published in a city. A comprehensive survey indicates that amongst the adult population of the city, 20% read A, 16% read B and 14% read C, but the survey also reveals that 8% read both A and B, 5% read both A and C, and 4% read both B and C, and that 2% read all three newspapers. A person is selected at random from the adult population of the city (that is, all adults are equally likely to be selected).

Use formal notation to express the following events in terms of A , B , C , and find their probabilities.

- (i) reads none of the newspapers

2 MARKS

- (ii) reads precisely one of the newspapers

3 MARKS

- (iii) reads at least A and B, **given** that they read at least one newspaper

5 MARKS

- 2 A dice game involves rolling two fair dice (each with six equally likely scores 1, 2, ..., 6) for which the two scores are probabilistically independent. The game is played as follows; if a combined score of 7 or 11 is obtained, the game is won; if a combined score of 2, 3, or 12 is obtained, the game is lost; if any other score is obtained, the dice are rolled again repeatedly until a combined score of 7 is obtained (when the game is won) or until the combined score rolled on the very first roll is obtained (in which case the game is lost).

- (i) Let E_j denote the event that a combined score of j is obtained on the very first roll, for $j = 2, \dots, 12$. Find $P(E_j)$, for $j = 2, \dots, 12$.

2 MARKS

- (ii) Let F_n denote the event that the game is won on roll n , for $n = 2, 3, 4, \dots$, and let W denote the event that the game is won. By considering the partition of F_n

$$F_n \equiv \bigcup_{j=2}^{12} (F_n \cap E_j)$$

and using the **Theorem of Total Probability**, show that the probability of winning the game overall can be expressed as

$$P(W) = P(F_1) + \sum_{n=2}^{\infty} \sum_{j=2}^{12} P(F_n \cap E_j) = P(F_1) + \sum_{j=2}^{12} \sum_{n=2}^{\infty} P(F_n \cap E_j)$$

3 MARKS

- (iii) For each *appropriate* j find the conditional probability $P(F_n | E_j)$, and hence find $P(W)$.

5 MARKS

Hint: here the sample space S is the set of sequences of rolls that could occur. To win on roll $n \geq 2$, given that a combined score of j was obtained on the very first roll, a sequence of $n - 2$ rolls of scores neither 7 nor j must occur next, followed finally by a score of 7 on the n th roll.