

MATH 323 - EXERCISES 8

NOT FOR ASSESSMENT.

TRANSFORMATIONS

- 1 Suppose that Y is a continuous random variable with range $\mathcal{Y} = [0, 1]$, and pdf f_Y specified by

$$f_Y(y) = 2(1 - y) \quad 0 \leq y \leq 1$$

and zero otherwise. Find the probability distributions of random variables U_1, U_2 and U_3 defined respectively by

$$U_1 = 2Y - 1 \quad U_2 = 1 - 2Y \quad U_3 = Y^2$$

In each case, find the range of values that the new variable can take, and its pdf or cdf.

- 2 The annual profit (in millions of dollars) of a manufacturing company is a function of product demand. If Y is the continuous random variable corresponding to the demand in a given year, then the annual profit is also a continuous random variable, U say, where $U = 2(1 - e^{-2Y})$.

If Y has an Exponential distribution with parameter $\beta = 1/6$, find the expected annual profit.

- 3 The random variable Y has a continuous uniform distribution on the interval $[-1, 1]$. Find the pdfs of the transformed random variables

$$U = |Y| \quad Z = Y^2$$

- 4 If Y is **any** continuous random variable with distribution function F_Y , show that

- (a) $U = F_Y(Y)$ has a continuous Uniform distribution on $[0, 1]$, and
- (b) $Z = -\ln U$ has an Exponential distribution.

- 5 If Y is a continuous random variable with pdf specified by

$$f_Y(y) = \alpha\beta y^{\alpha-1} e^{-\beta y^\alpha} \quad y > 0$$

and zero otherwise, for parameters $\alpha, \beta > 0$, then Y has a *Weibull* distribution. Show that $U = Y^\alpha$ has an Exponential distribution.

- 6 Suppose that random variable Y has a standard normal distribution.

- (a) Find the cdf of random variable $X = Y^2$ in terms of the standard normal cdf, denoted Φ . Recall that for the cdf of X , we have

$$P(X \leq x) = P(Y^2 \leq x) = P(|Y| \leq \sqrt{x})$$

- (b) Find the pdf of X , f_X .
- (c) Identify (by name) the probability distribution of X .

- 7 Suppose that Y_1 and Y_2 are independent and identically distributed random variables, each having a standard normal distribution. Let random variable V be defined by

$$V = Y_1^2 + Y_2^2$$

Find the pdf of V .