

MATH 323 - ASSIGNMENT 3

SOLUTIONS

1. By direct calculation, or from the distributions sheet, we have $F(y) = 0$ for $y \leq 0$ and

$$F(y) = 1 - e^{-y/\beta} \quad y > 0.$$

(a) (i) $P(Y \leq 2) = 1 - e^{-2} = 0.8647$. 2 MARKS

(ii) $P(Y > 4) = 1 - (1 - e^{-2}) = e^{-2} = 0.1353$. 2 MARKS

(iii) We have

$$P(Y > 4|Y > 2) = \frac{P((Y > 4) \cap (Y > 2))}{P(Y > 2)} = \frac{P(Y > 4)}{P(Y > 2)} = \frac{e^{-4}}{e^{-2}} = e^{-2} = 0.1353.$$

2 MARKS

(iv) This probability is zero, as Y is continuous. 2 MARKS

(b) From first principles, for $x > 0$

$$F_X(x) = P(X \leq x) = P(3Y \leq x) = P(Y \leq x/3) = 1 - e^{-x/3\beta}$$

so $X \sim \text{Exponential}(3\beta)$. 2 MARKS

2. (a) (i) $P(Y \leq 2) = 0.9772$. 2 MARKS

(ii) $P(Y > 1) = 1 - P(Y \leq 1) = 1 - P((Y - 1)/1 \leq (1 - 1)/1) = 1 - P(Z \leq 0) = 1/2$. 2 MARKS

(iii) Whatever the distribution of Y

$$P(Y > 0|Y > 2) = \frac{P((Y > 0) \cap (Y > 2))}{P(Y > 2)} = \frac{P(Y > 2)}{P(Y > 2)} = 1.$$

2 MARKS

(iv) By Axiom III,

$$P(Y > 2 \text{ or } Y < -1) = P(Y > 2) + P(Y < -1) = 1 - P(Y \leq 2) + P(Y \leq -1)$$

and if Z denotes a standard Normal random variable

$$P(Y \leq 2) = P(Y/2 \leq 2/2) = P(Z \leq 1) = 0.8413$$

$$P(Y \leq -1) = P(Y/2 \leq -1/2) = P(Z \leq -0.5) = P(Z > 0.5) = 1 - P(Z \leq 0.5) = 0.3085$$

by symmetry of the pdf. Therefore

$$P(Y > 2 \text{ or } Y < -1) = 1 - 0.8413 + 0.3085 = 0.4672.$$

2 MARKS

(b) We have that for $x \in \mathbb{R}$,

$$F_X(x) = P(X \leq x) = P(2Y - 1 \leq x) = P(Y \leq (x + 1)/2) = F_Y((x + 1)/2)$$

and hence by differentiation

$$\begin{aligned} f_X(x) &= \frac{1}{2} f_Y((x + 1)/2) = \frac{1}{2\sigma\sqrt{2\pi}} \exp \left\{ -\frac{1}{2\sigma^2} ((x + 1)/2 - \mu)^2 \right\} \\ &= \frac{1}{2\sigma\sqrt{2\pi}} \exp \left\{ -\frac{1}{8\sigma^2} (x - (2\mu - 1))^2 \right\} \end{aligned}$$

so therefore $X \sim \text{Normal}(2\mu - 1, 4\sigma^2)$.

2 MARKS

3. (a) We need the limit of $F_Y(y)$ to be 1 as $y \rightarrow \infty$, so $c = 1$.

2 MARKS

(b) $P(Y > 1) = 1 - F_Y(1) = 1 - \exp\{-e^{-1}\} = 0.3078$.

3 MARKS

(c) X takes values on $\mathbb{R}^+$, and for $x > 0$

$$\begin{aligned} F_X(x) &= P(X \leq x) = P(e^{-Y} \leq x) = P(Y \geq -\ln x) \\ &= 1 - P(Y < -\ln x) \\ &= 1 - \exp\{-e^{\ln x}\} \\ &= 1 - e^{-x} \end{aligned}$$

and so $X \sim \text{Exponential}(1)$.

5 MARKS