

$$\frac{1}{y_1(y_1+1)} - \frac{1}{(y_1+1)(y_1+2)}$$

$$+ \frac{1}{(y_1+1)(y_1+2)} - \frac{1}{(y_1+2)(y_1+3)}$$

$$+ \dots$$

$$[E(Y_1)]$$

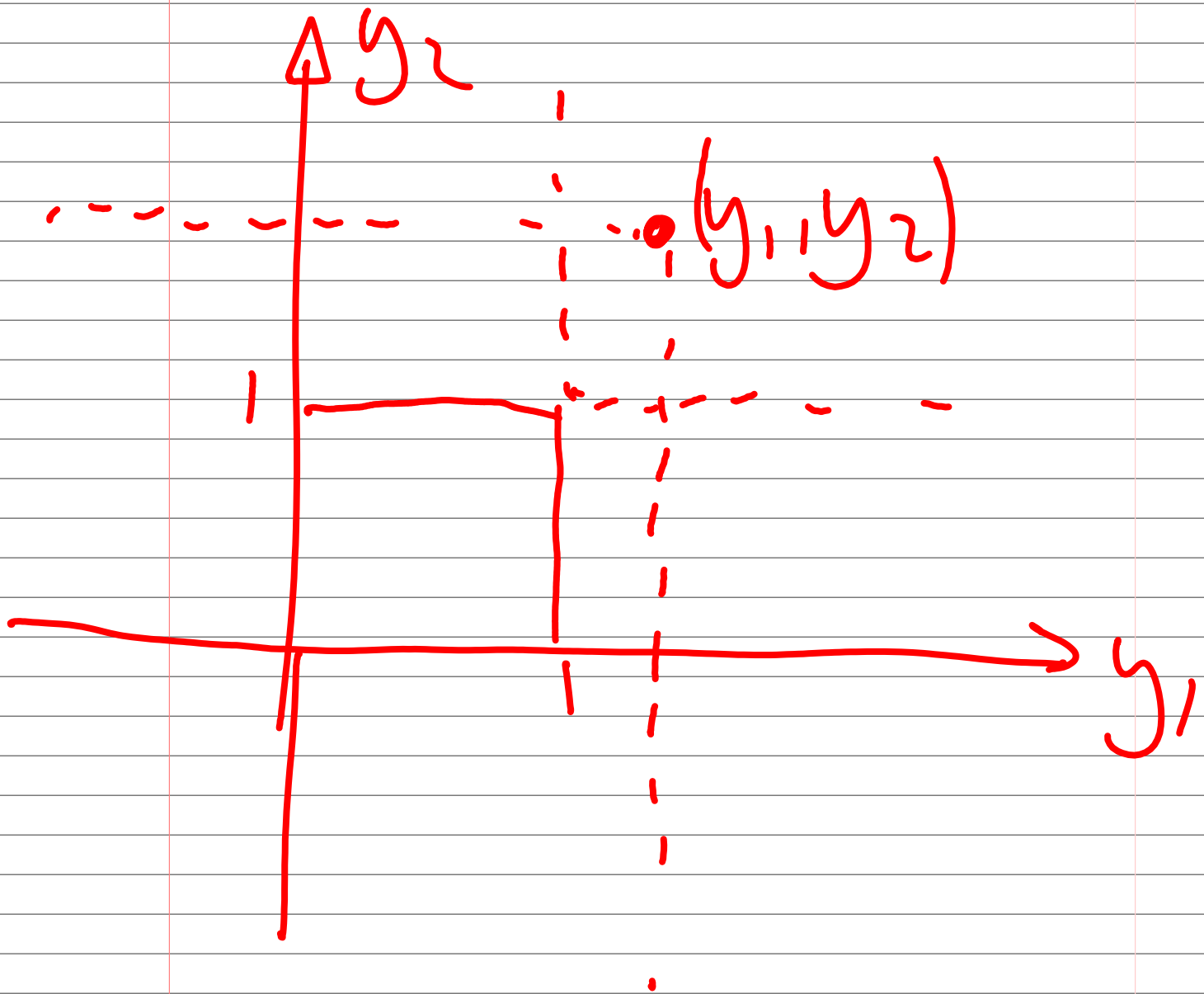
$$= \sum_{y_1=1}^{\infty} y_1 \cdot p(y_1)$$

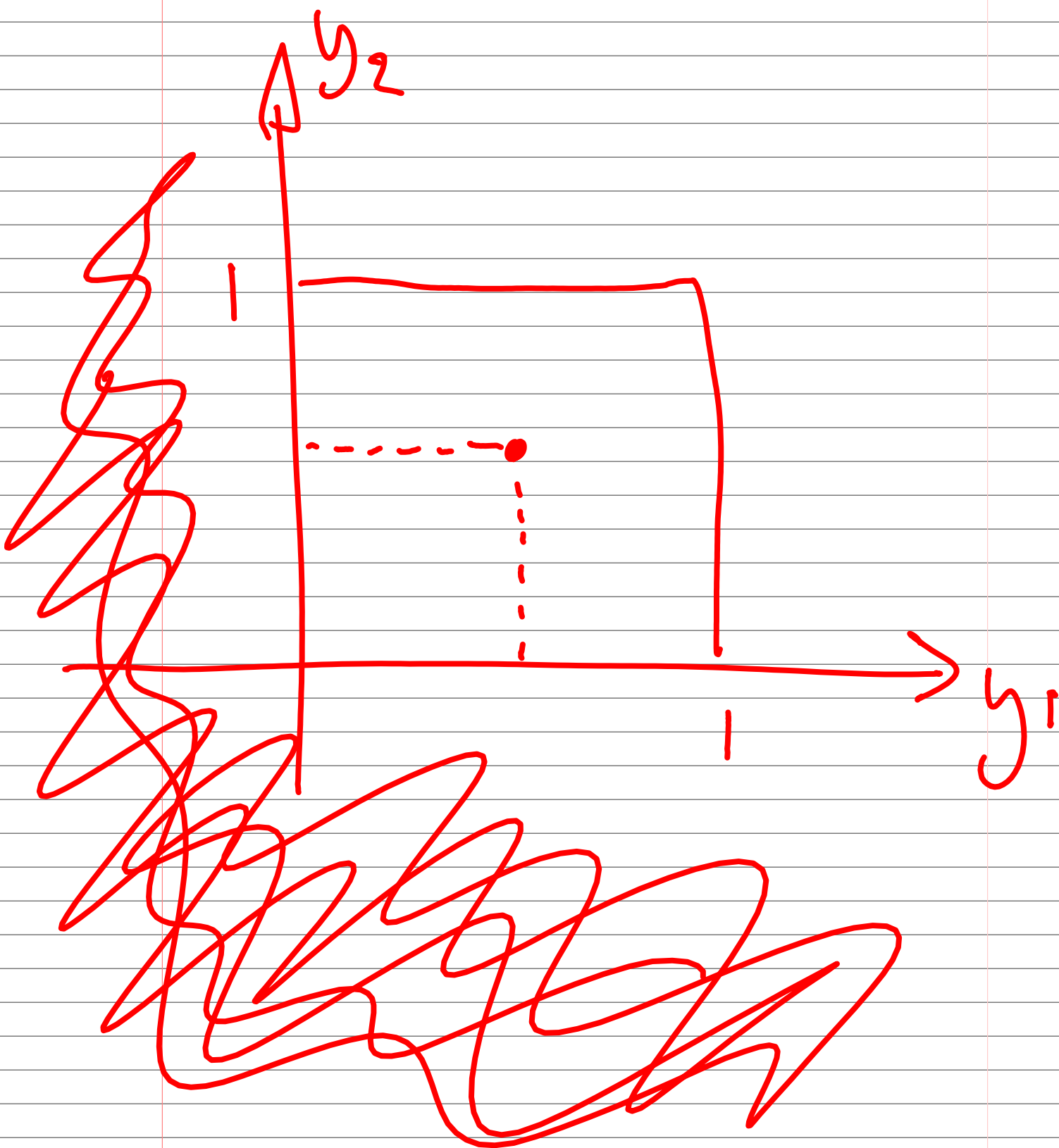
$$= \sum_{y_1=1}^{\infty} y_1 \cdot \frac{1}{y_1(y_1+1)}$$

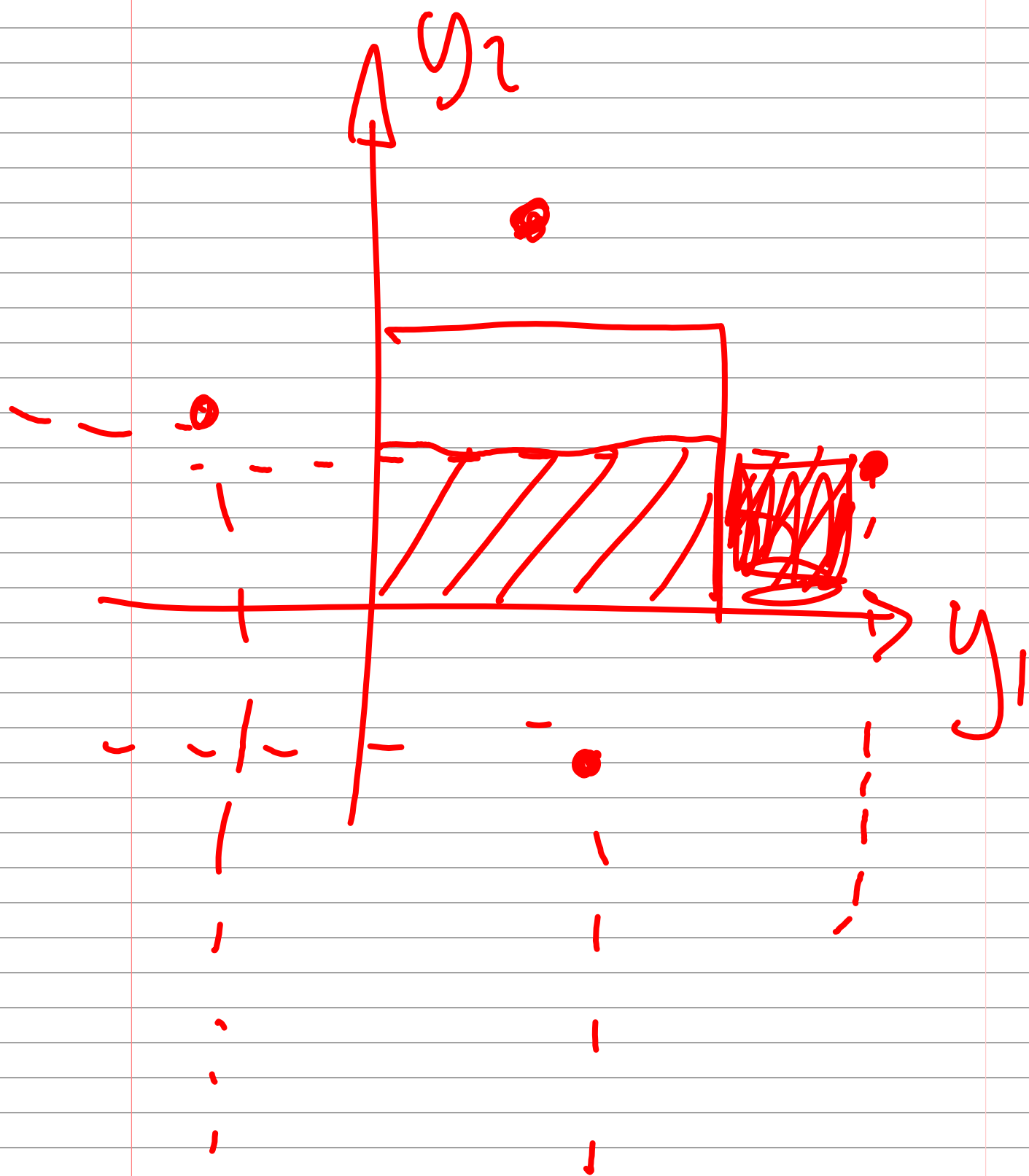
$$= \sum_{y_1=1}^{\infty} \frac{1}{y_1+1} = \infty$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$$

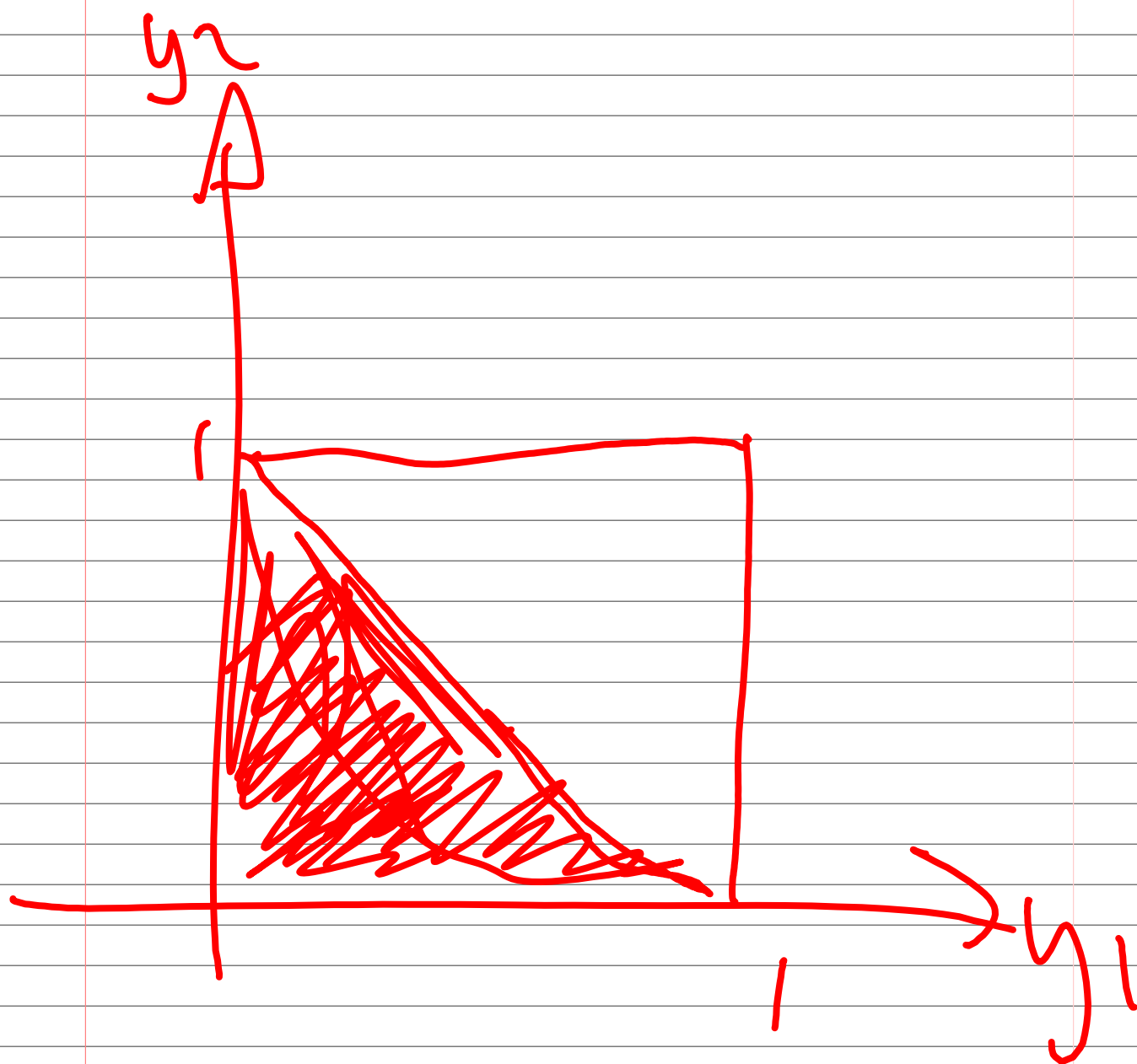
$$F_{Y_1, Y_2}(y_1, y_2) = P[Y_1 \leq y_1, Y_2 \leq y_2]$$



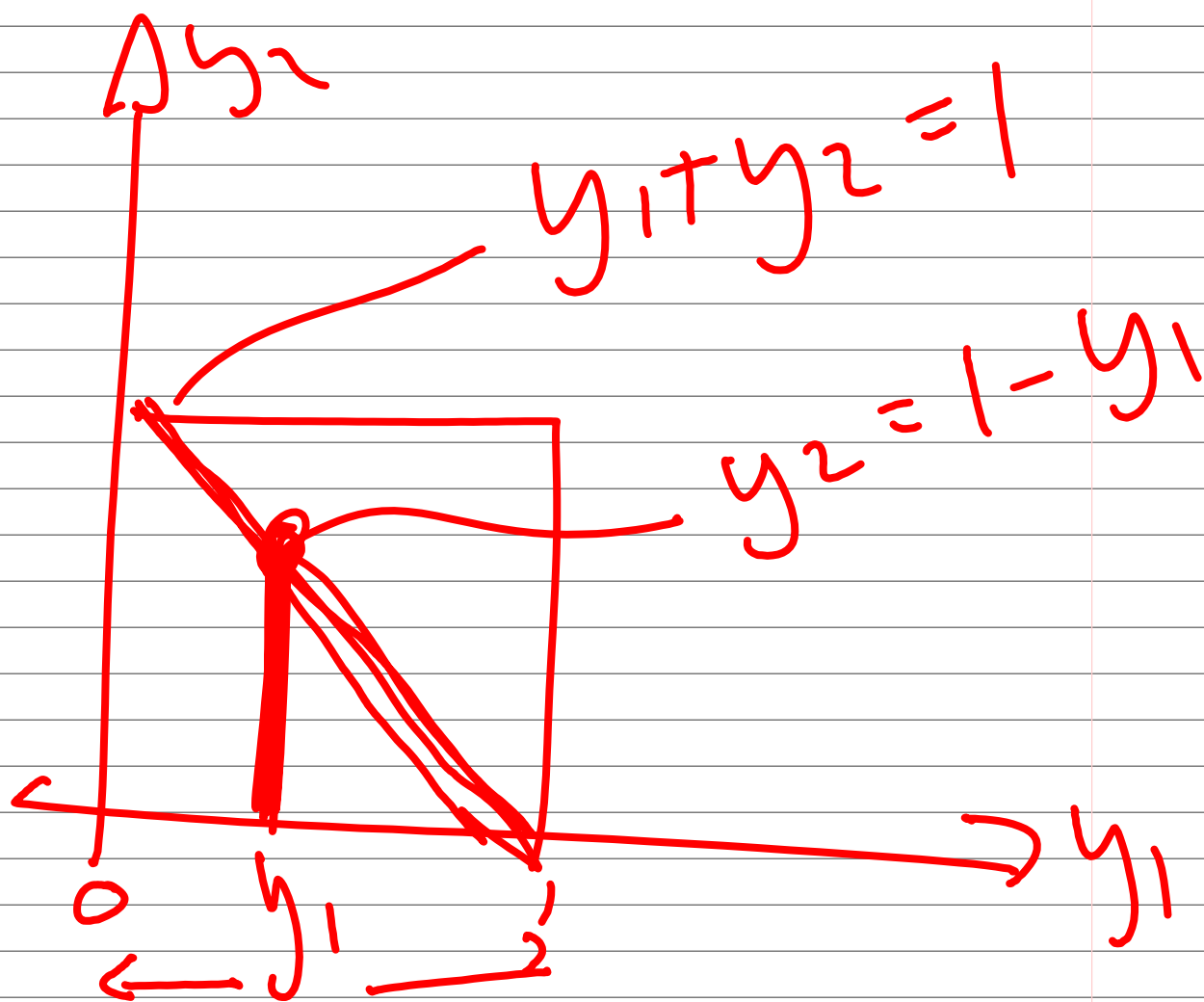




$$P[Y_1 + Y_2 < 1]$$



$$\int_0^1 \left\{ \int_0^{1-y_1} 4y_1 y_2 dy_2 \right\} dy_1$$

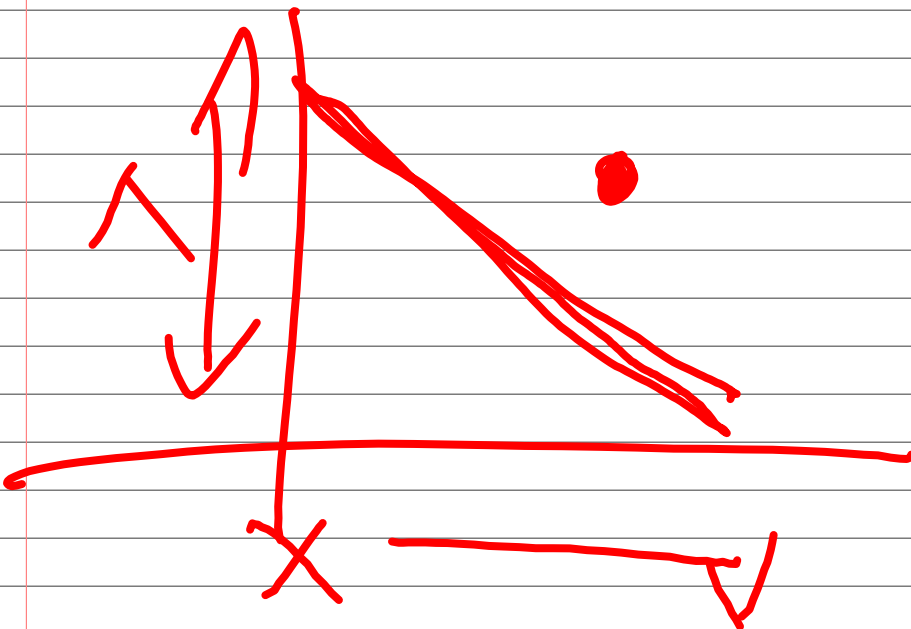


$$f_{Y_1, Y_2}(y_1, y_2) = c(y_1 + y_2)$$

$$0 < y_1 < 1$$

$$0 < y_2 < 1$$

zero otherwise



$$P[Y_1 + Y_2 > y]$$

$$P[Y_1 > Y_2]$$

$$P[Y_1 - Y_2 > 0]$$

$$\iint_A f_{y_1, y_2}(y_1, y_2) dy_2 dy_1$$

$$A: \{ (y_1, y_2) : y_2 < y_1 \}$$



$$\left(\frac{1}{1-\beta t} \right)^\alpha$$

$$Y_1 \sim \text{Ga}(\alpha_1, \beta)$$

$$Y_2 \sim \text{Ga}(\alpha_2, \beta)$$

$$Y = Y_1 + Y_2$$

$$m_Y(t) = m_{Y_1}(t) m_{Y_2}(t)$$

$$\left(\frac{1}{1-\beta t}\right)^{\alpha_1} \left(\frac{1}{1-\beta t}\right)^{\alpha_2}$$

$$= \left(\frac{1}{1-\beta t}\right)^{\alpha_1 + \alpha_2}$$

$$\Rightarrow Y \sim \text{Ga}(\alpha_1 + \alpha_2, \beta)$$

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$$\frac{d}{dy} \left\{ \int_0^y \int_0^y -dt dy_1 \right\}$$

$$\frac{d}{dy} \int_0^y \{g(y_1, y)\} dy_1$$

$$\int_{-\infty}^{\infty} f_{Y_1, Y_2}(y_1, y - y_1) dy_1$$

$$0 \leq y_1 \leq y - y_1 \leq 1$$

$$(i) \quad y_1 \leq y - y_1, \quad y > 2y_1 \\ \frac{y}{2} > y_1$$

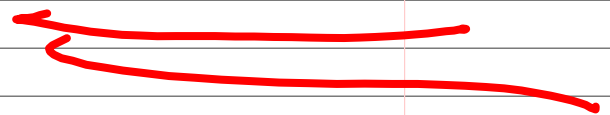
$$(ii) \quad y - y_1 \leq 1 \\ \therefore y_1 > y - 1$$

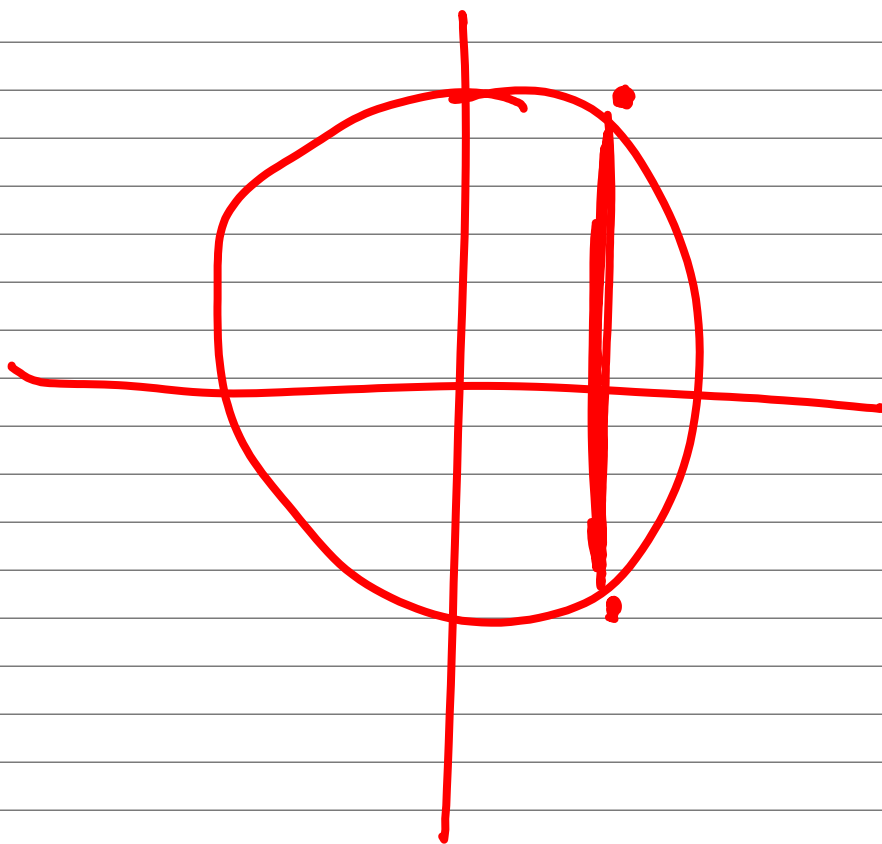
$$\frac{y}{2} \geq y_1$$

$$y_1 \geq y - 1$$

$$y_1 \geq 0 \quad y_1 \leq 1$$

$$\max\{0, y-1\} \leq y_1 \leq \min\left\{\frac{y}{2}, 1\right\}.$$





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