

$$p(y_1, y_2)$$

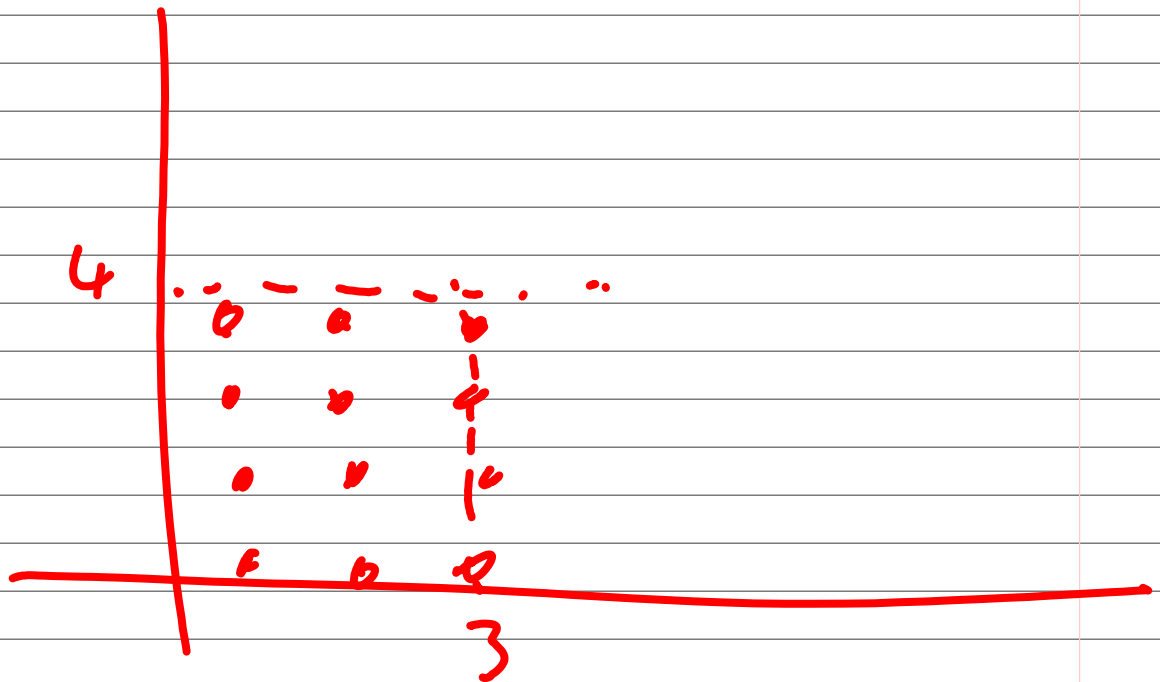
$$= c \exp \left\{ 1 + \frac{(y_1 - 4)(y_2 - 2)}{10} \right\}$$

$$0 \leq y_1 \leq 8$$

$$0 \leq y_2 \leq 6$$

$$\frac{1}{c} = \sum_{y_1=0}^8 \sum_{y_2=0}^6 \exp \left\{ 1 + \frac{(y_1 - 4)(y_2 - 2)}{10} \right\}$$

$$P(Y_1 \leq 3, Y_2 \leq 4)$$



$$P(Y_1 = y_1 | Y_2 = y_2)$$

$$= \frac{P(Y_1 = y_1, Y_2 = y_2)}{P(Y_2 = y_2)}$$

$$P(Y_2 = y_2 | Y_1 = y_1)$$

$$= \frac{P(Y_1 = y_1, Y_2 = y_2)}{P(Y_1 = y_1)}$$

$$P(Y_1 \leq y_1, Y_2 \leq y_2)$$

$$= F_{Y_1, Y_2}(y_1, y_2)$$

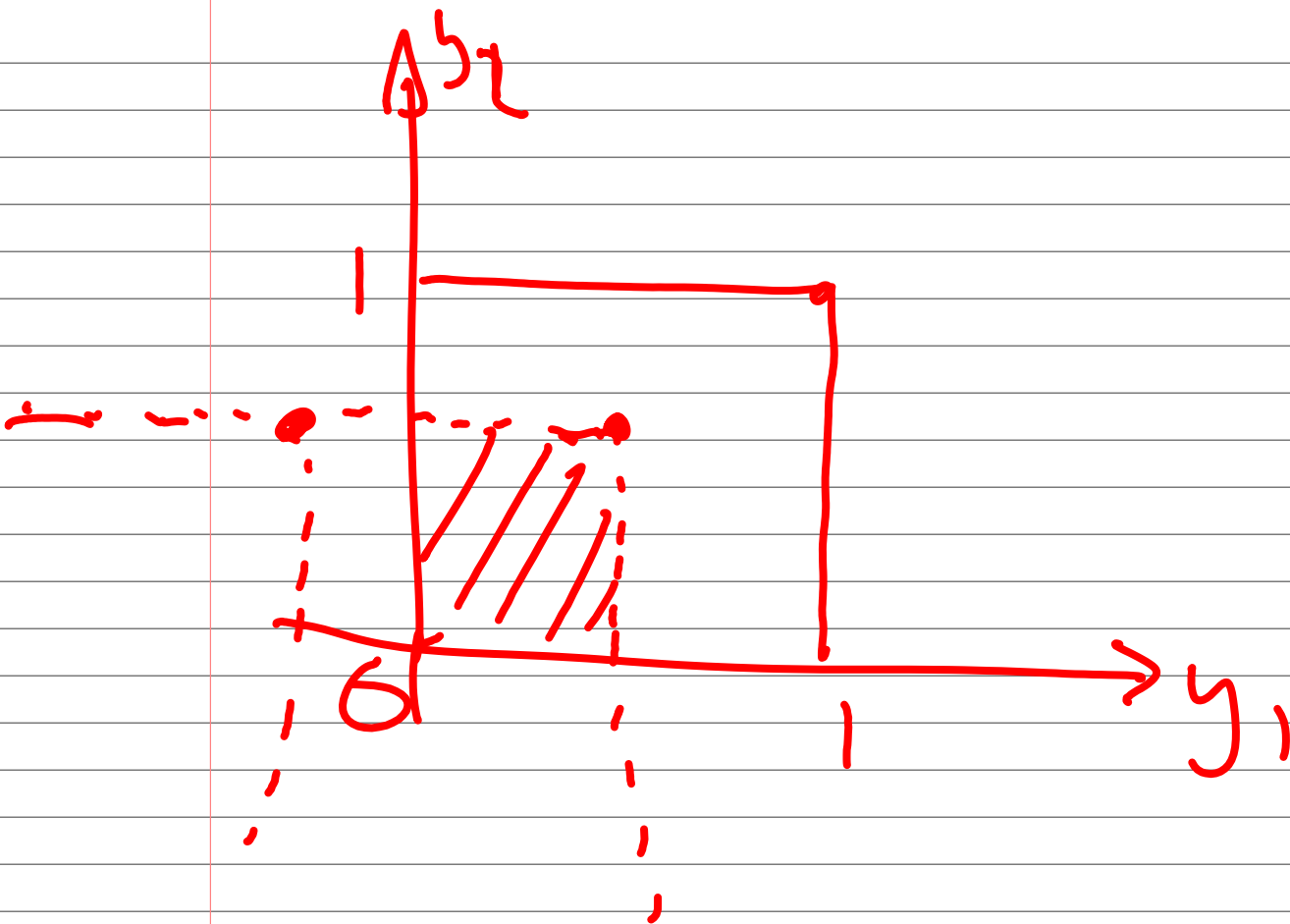
— continuous in
 y_1, y_2

$$\lim_{y_1 \rightarrow -\infty} F_{Y_1, Y_2}(y_1, y_2)$$

$$= P(Y_1 \leq -\infty, Y_2 \leq y_2)$$

$$= 0$$

as $Y_1 \leq -\infty$
cannot happen!



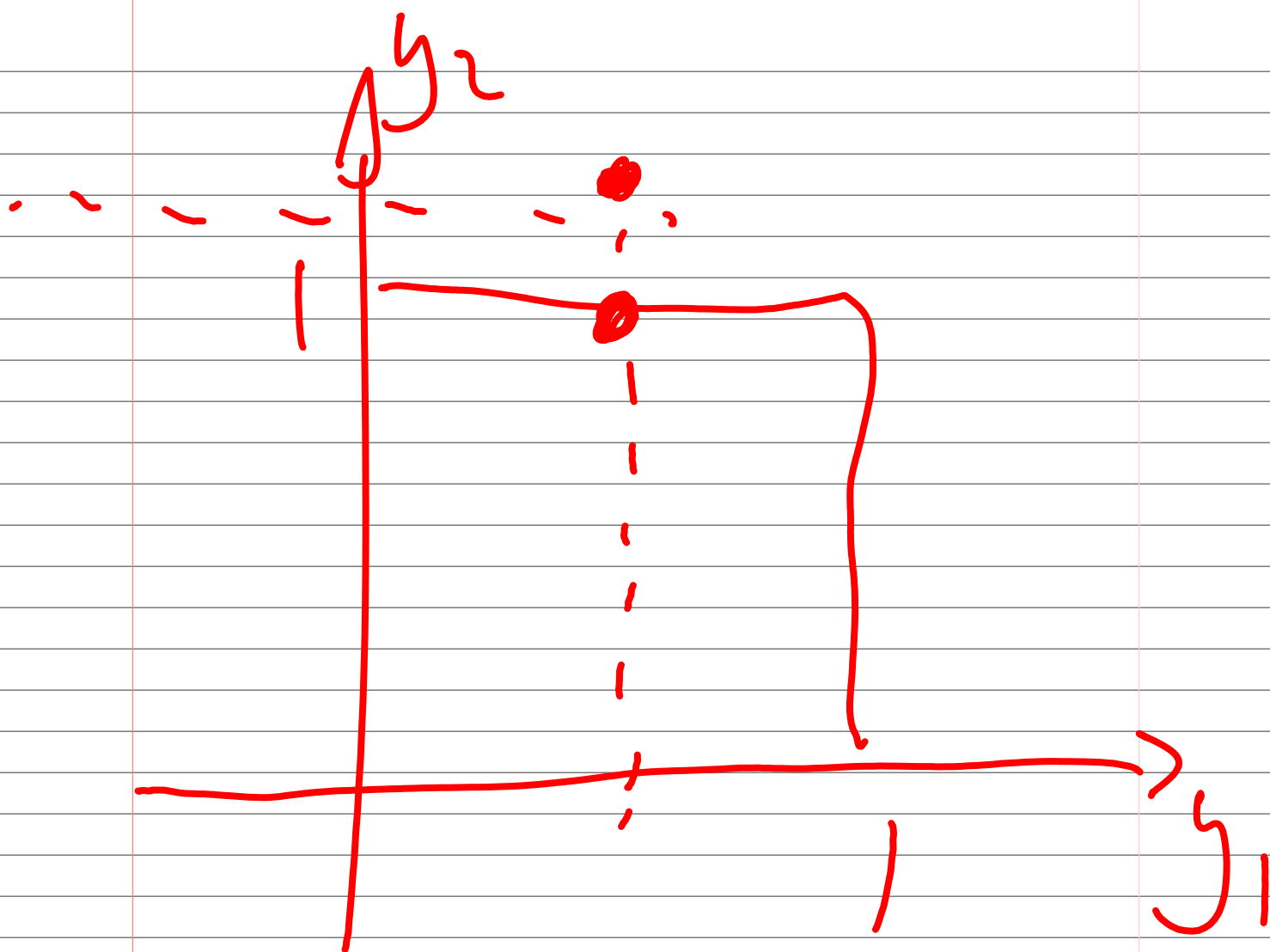
$$P(Y_1 \leq y_1, Y_2 \leq y_2)$$

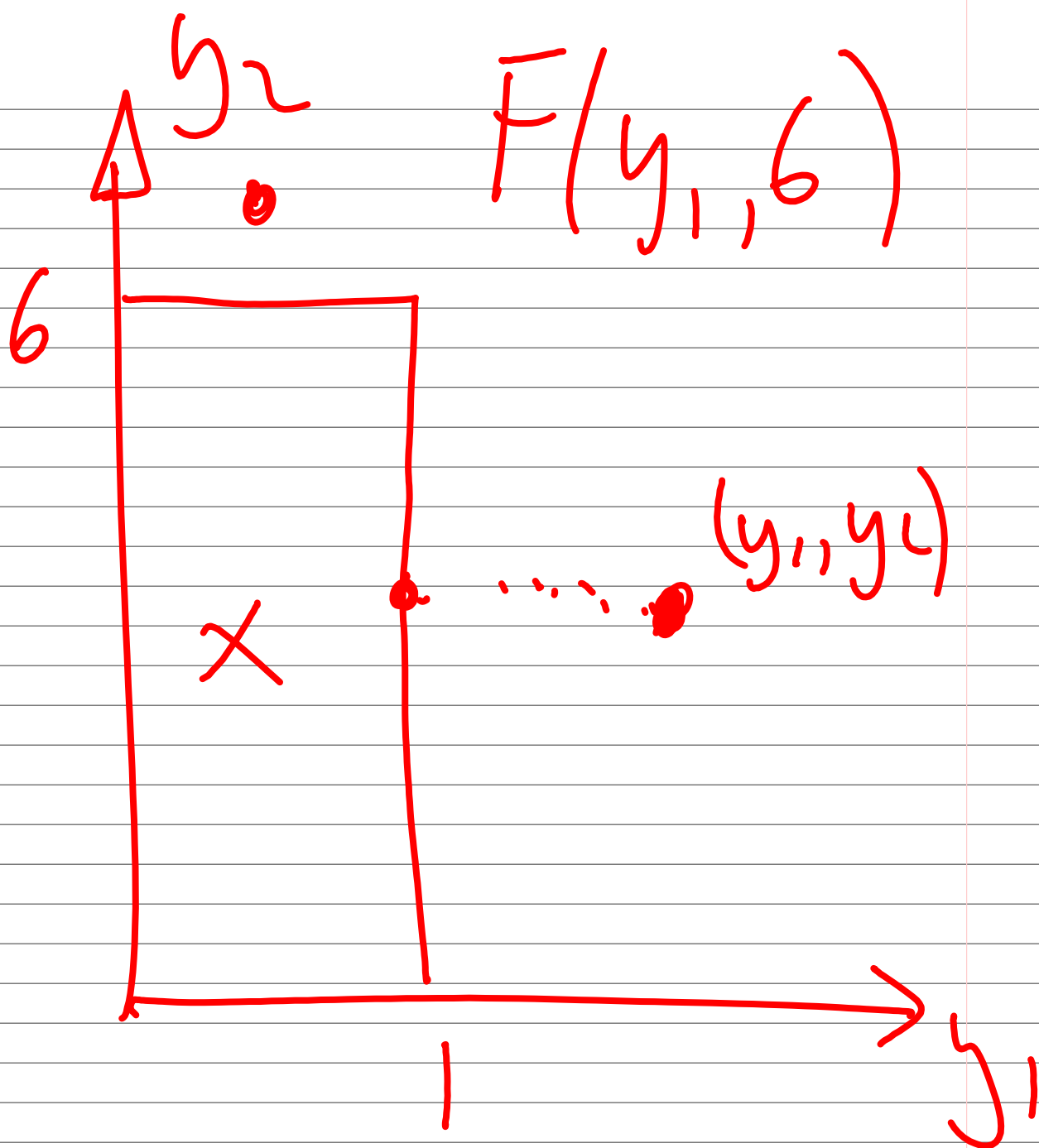
$$\int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f_{Y_1, Y_2}(t_1, t_2) dt_2 dt_1$$

$$P(Y_1 \leq y_1, Y_2 \leq \infty)$$

$$= P(Y_1 \leq y_1)$$

$$= F_{Y_1}(y_1)$$





$$F(y_1, y_2)$$

$$= F(1, y_2)$$

$$F_{Y_1, Y_2}(y_1, y_2)$$

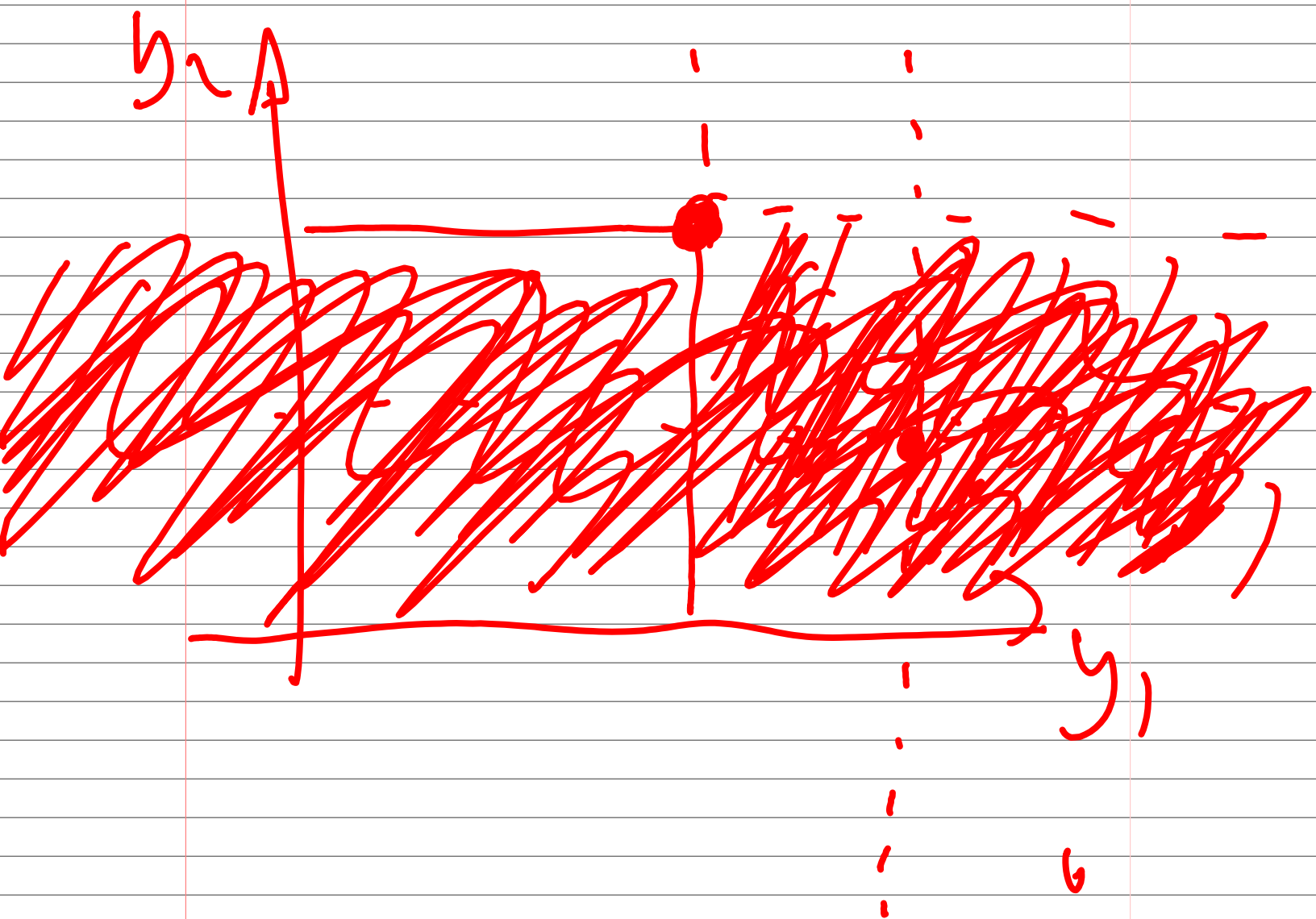
$$= P(Y_1 \leq y_1, Y_2 \leq y_2)$$

$$P(Y_1 > 1, Y_2 > 1)$$

$$= 1 - P(Y_1 \leq 1 \text{ or } Y_2 \leq 1)$$

$$= 0$$

$$P(Y_1 > 1, Y_2 \leq 1)$$



$$P(Y_1 > 1, Y_2 \leq 1)$$

$$= \underline{P(Y_2 \leq 1)} \underline{P(Y_1 > 1 | Y_2 \leq 1)}$$

$$\Rightarrow (Y_1 > 1) \cap (Y_2 \leq 1)$$

$$= (Y_2 \leq 1) - (Y_2 \leq 1, Y_1 \leq 1)$$

$$(Y_2 \leq 1)$$

$$= (Y_2 \leq 1, Y_1 \leq 1)$$

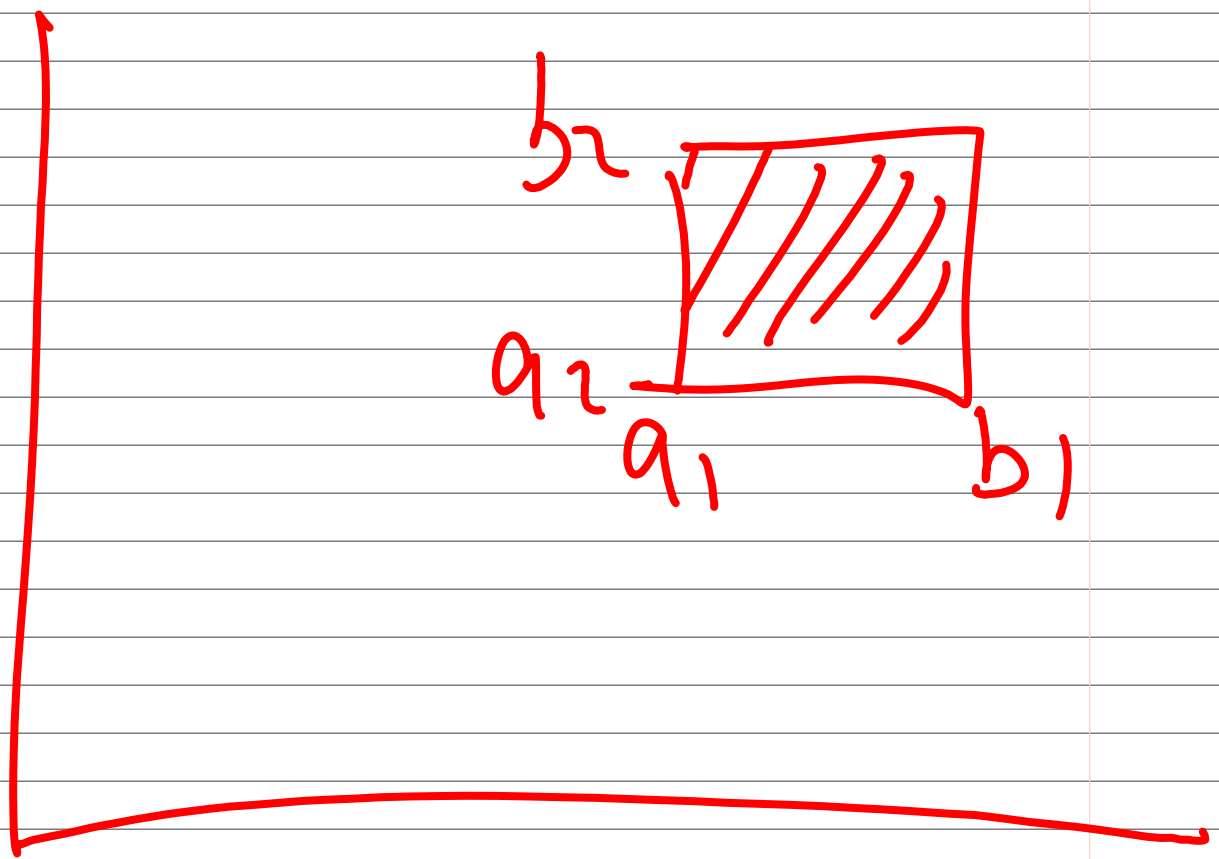
$$\cup (Y_2 \leq 1, Y_1 > 1)$$

$$\Rightarrow P(Y_2 \leq 1)$$

$$= P(Y_2 \leq 1, Y_1 \leq 1) + P(Y_2 \leq 1, Y_1 > 1)$$

$$P(a_1 \leq y_1 \leq b_1,$$

$$a_2 \leq y_2 \leq b_2)$$



$$P(a_1 \leq Y_1 \leq b_1)$$

$$= F_{Y_1}(b_1) - F_{Y_1}(a_1)$$



$$F_{\gamma_1, \gamma_2}(b_1, b_2) - F_{\gamma_1, \gamma_2}(a_1, b_2) - F_{\gamma_1, \gamma_2}(b_1, a_2)$$

$$P(a_1 \leq Y_1 \leq b_1, a_2 \leq Y_2 \leq b_2)$$

$$= F_{Y_1, Y_2}(b_1, b_2)$$

$$- F_{Y_1, Y_2}(a_1, b_2)$$

$$- F_{Y_1, Y_2}(b_1, a_2)$$

$$+ F_{Y_1, Y_2}(a_1, a_2)$$

$$f_{y_1, y_2}(y_1, y_2)$$

$$= \frac{(1 + y_1 y_2)}{(1 + y_2/2)} \quad 0 \leq y_1 \leq 1$$

when $0 \leq y_2 \leq 1$

$$= ay_1 + b$$

$$a = \frac{y_2}{1 + y_2/2}$$

$$b = \frac{1}{1 + y_2/2}.$$

$$F_{y_1, y_2}(y_1, y_2) = cy_1 y_2$$

$$0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1$$

(1) $c = 1$ as
we need

$$F_{y_1, y_2}(1, 1) = 1$$

$$(ii) F_{Y_1}(y_1) = y_1$$

$$0 \leq y_1 \leq 1$$

$$(iii) f_{Y_1}(y_1) = 1$$

$$0 \leq y_1 \leq 1$$

zero o.w.

$$(iv) F_{Y_1, Y_2}(y_1, y_2) = y_1 y_2$$

$\Rightarrow$

$$f_{Y_1, Y_2}(y_1, y_2) = 1$$

$$0 \leq y_1 \leq 1$$

$$0 \leq y_2 \leq 1$$

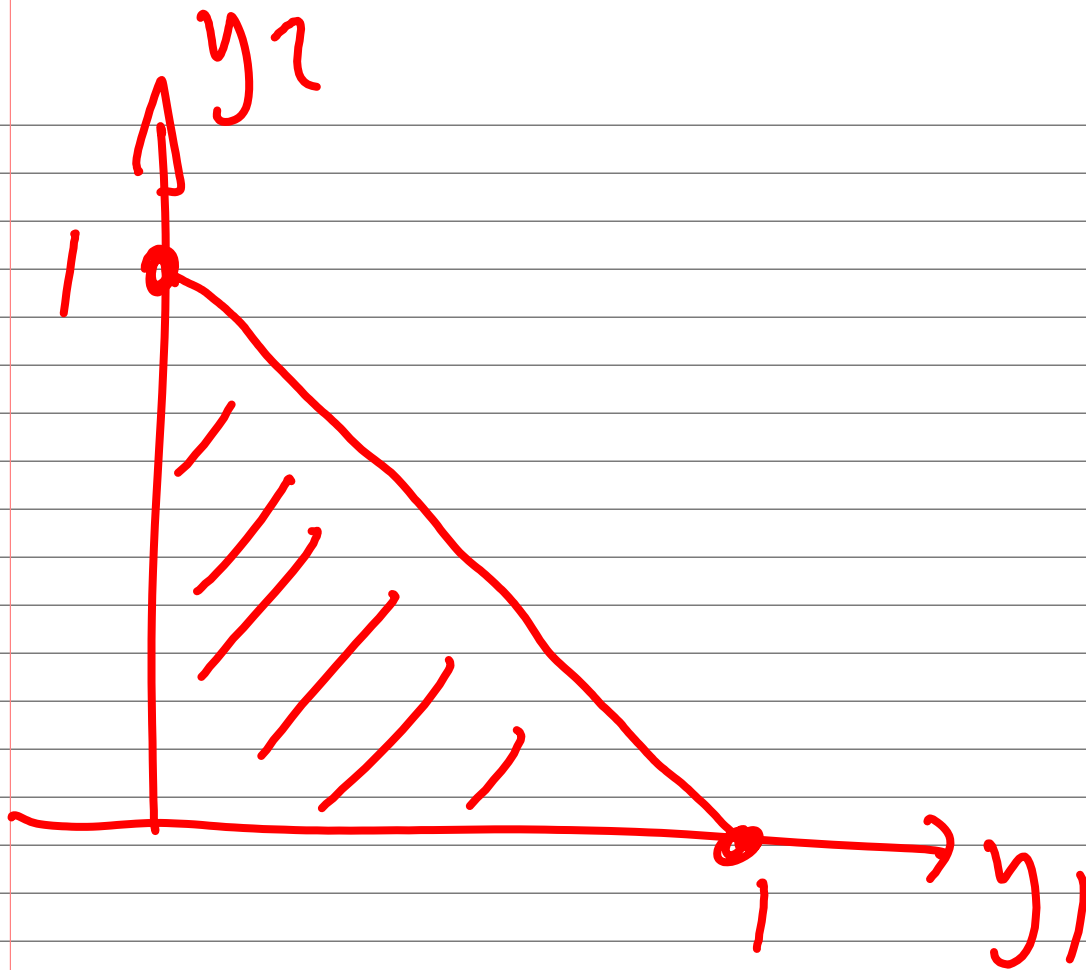
(v) Y_1, Y_2 independent?

Yes:

$$F_{Y_1, Y_2}(y_1, y_2) = F_{Y_1}(y_1) F_{Y_2}(y_2)$$

$$f_{Y_1, Y_2}(y_1, y_2) = f_{Y_1}(y_1) f_{Y_2}(y_2)$$

$$\forall y_1, y_2$$



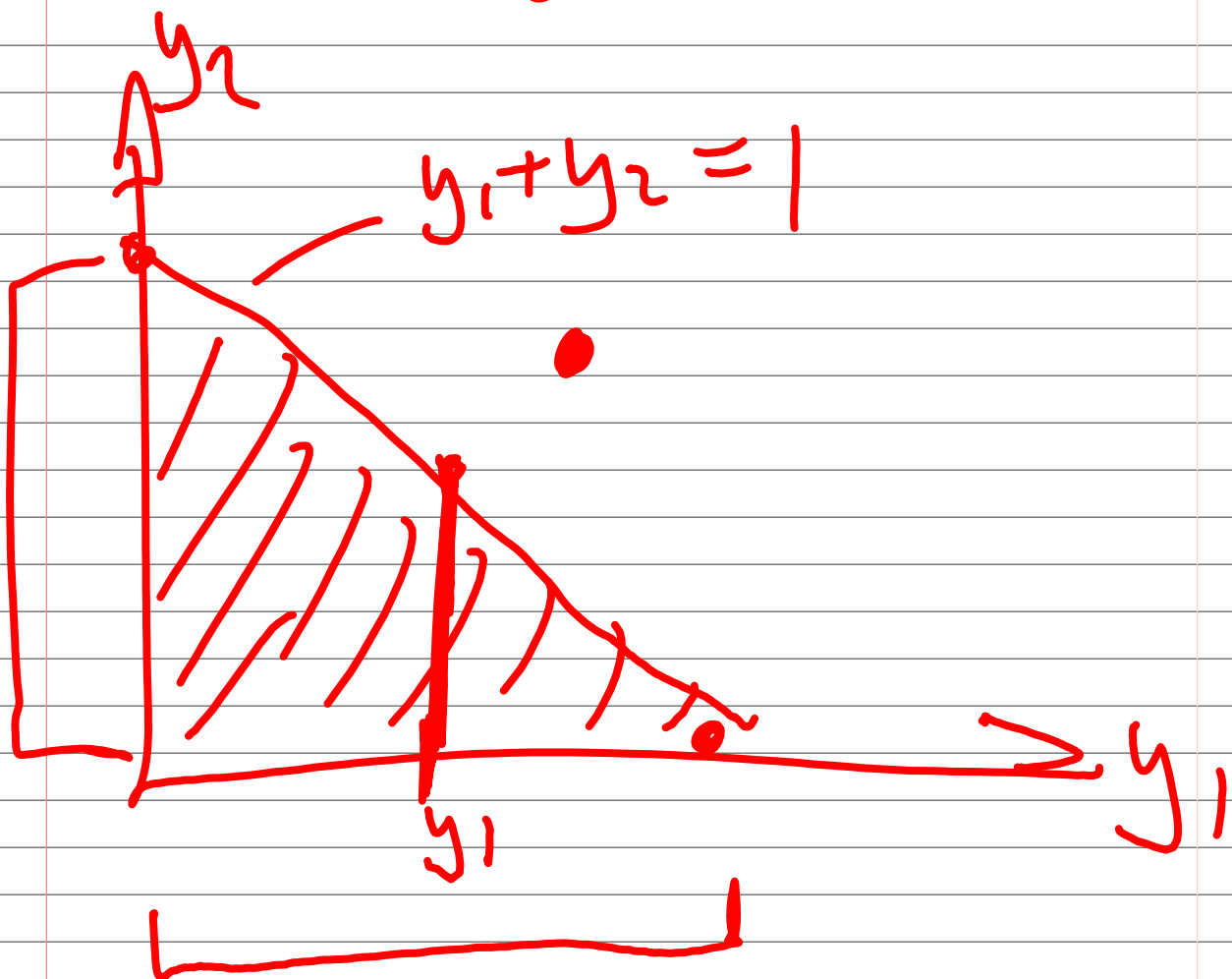
$$f_{Y_1, Y_2}(y_1, y_2) = c = 2$$

$$0 \leq y_1 + y_2 \leq 1$$

zero o.w.

$$f_{Y_1}(y_1) = \int_{-\infty}^{\infty} f_{Y_1, Y_2}(y_1, y_2) dy_2$$

$$= \int_0^{1-y_1} 2 dy_2$$



$$= \begin{bmatrix} 2y_2 \end{bmatrix}_0^{1-y_1}$$

$$= 2(1-y_1) \quad 0 \leq y_1 \leq 1$$

zero otherwise.

$$f_{Y_2}(y_2) = 2(1-y_2) \\ 0 \leq y_2 \leq 1$$

$$[0,1] \times [0,1]$$

$\uparrow \qquad \qquad \uparrow$

$$(y_1, y_2)$$

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