

$$P(y) = (1-p)^{y-1} p$$

$$y \in \{1, 2, 3, \dots\}$$

and zero otherwise

1.

$$f(y) = cy^2 + y$$

$0 < y \leq 1$
— and zero o.w.

$$\int_{-\infty}^{\infty} f(y) dy = 1$$

$$\Rightarrow \int_0^1 (cy^2 + y) dy = 1$$

(b)

$$F(y) = \int_{-\infty}^y f(t) dt$$

$$= \int_0^y (ct^2 + t) dt$$

$$0 \leq y \leq 1$$

$$= \left[\frac{ct^3}{3} + \frac{t^2}{2} \right]_0^y$$

$$= \frac{cy^3}{3} + \frac{y^2}{2} \quad 0 \leq y \leq 1$$

$$\text{But } F(1) = 1$$

$$\Rightarrow \frac{c}{3} + \frac{1}{2} = 1$$

$$\therefore 2c + 3 = 6$$

$$\therefore c = 1.5$$

$$(c) P(Y < \frac{1}{2})$$

$$\equiv P(Y \leq \frac{1}{2})$$

$$= F(1/2)$$

$$(d) P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(if $P(B) > 0$)

$$P(Y > \frac{1}{2} | Y > \frac{1}{4})$$

$$P(Y > 1/2 \mid Y > 1/4)$$

$$= \frac{P((Y > 1/2) \cap (Y > 1/4))}{P(Y > 1/4)}$$

$$P(Y > 1/2)$$

$$= \frac{P(Y > 1/2)}{P(Y > 1/4)}$$

$$= \frac{1 - F(1/2)}{1 - F(1/4)}$$

$$1 - F(1/4)$$

(e) S - 'total who finish in $< 10'$ '

$\Rightarrow$

$$S \sim \text{Binomial}(200, p)$$

$$n = 200$$

$$S = \sum_{i=1}^n Y_i$$

$$P(S \leq 3)$$

$$= P(S = 0) +$$

$$P(S = 1) +$$

$$P(S = 2) + P(S = 3)$$

2.

$$f(y) = e(1-y^2)$$

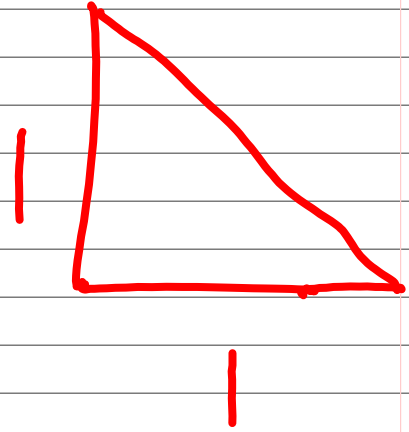
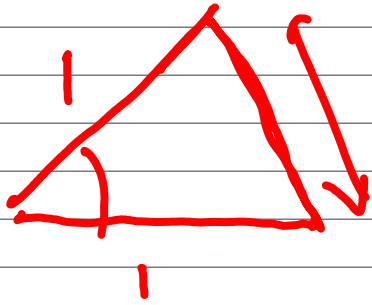
$$-1 \leq y \leq 1$$

and zero otherwise

$$F(y) = \int_{-\infty}^y f(t) dt$$

$$= \int_{-1}^y f(t) dt$$

3.



(a)

$$f(y) = cy(\pi - y) \quad 0 < y < \frac{\pi}{2}$$

(b)

$$X = \frac{1}{2} \sin Y$$

$$0 < Y < \frac{\pi}{2}$$

$$\Rightarrow 0 < X < \frac{1}{2}$$

$$F_X(x) \quad 0 < x < 1/2$$

$$F_X(x) = P(X \leq x)$$

$$= P\left(\frac{1}{2} \sin Y \leq x\right)$$

$$= P\left(Y \leq \sin^{-1}(2x)\right)$$

$$= F_y(\sin^{-1}(2x))$$

$$\therefore f_x(x) = \frac{d}{dx} \left\{ F_y(\sin^{-1}(2x)) \right\}$$

$$0 < x < 1/2$$

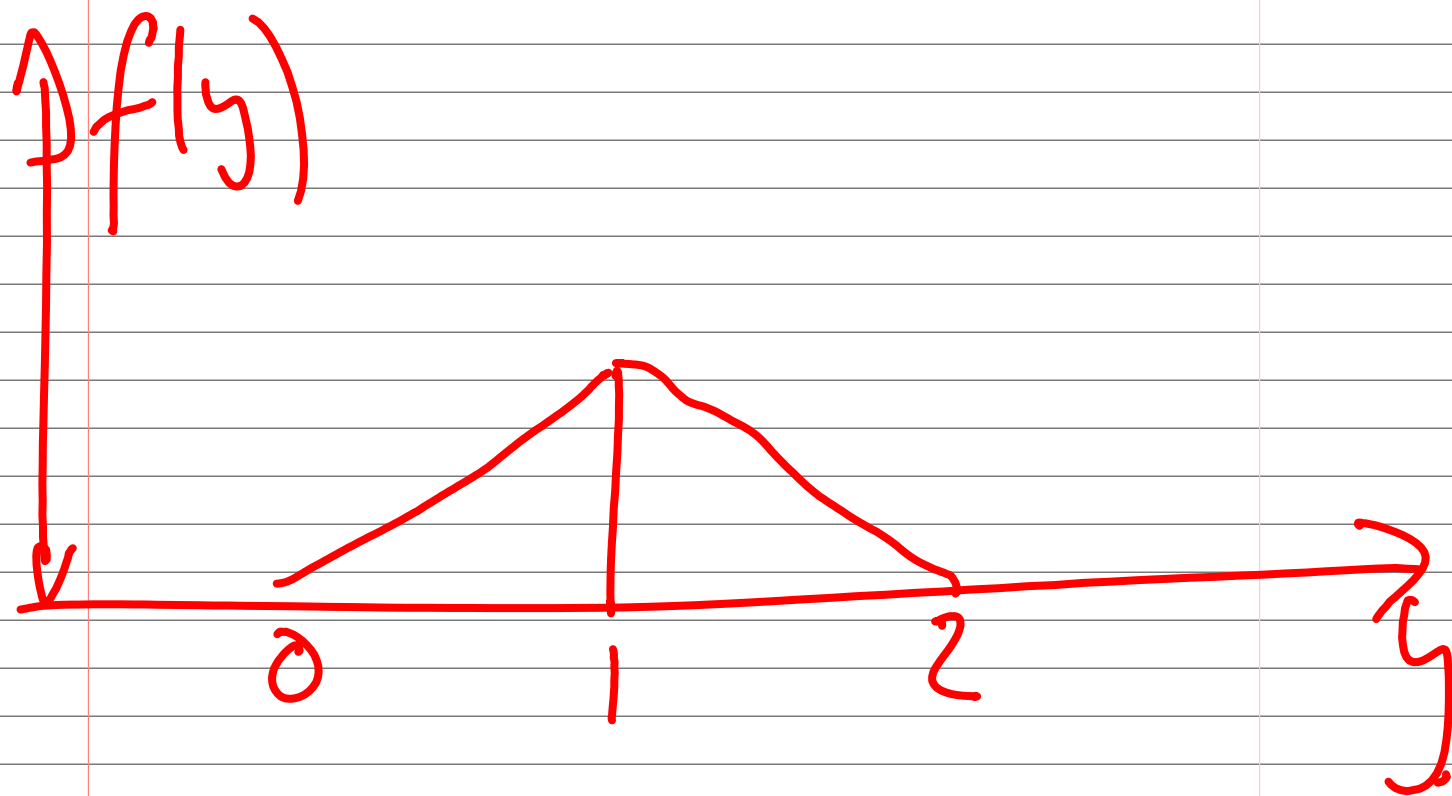
$$U = h(Y)$$

$$\Leftrightarrow Y = h^{-1}(U)$$

$$f_U(u) = f_Y(h^{-1}(u)) \left| \frac{dh^{-1}(u)}{du} \right|$$

'JACOBIAN'

$$5. f(y) = \begin{cases} y & 0 < y < 1 \\ 2-y & 1 \leq y < 2 \end{cases}$$



$$F(y) = \int_{-\infty}^y f(t) dt$$

$$= \int_0^y t dt \quad 0 < y < 1$$

$$\int_0^1 t dt + \int_1^y (2-t) dt$$

$$P(Y \leq y).$$

$$= \begin{cases} y^2/2 & 0 \leq y < 1 \\ \frac{1}{2} + \left[2t - \frac{t^2}{2} \right]^y & 1 \leq y \leq 2 \end{cases}$$

$$\rightarrow F(y)$$

$$P(0.8 < Y < 1.2) \\ = F(1.2) - F(0.8)$$

$$6. A : \text{Normal}(\mu_A, \sigma^2)$$

$$B : \text{Normal}(\mu_B, \sigma^2)$$

$$P(A) = 0.3$$

$$P(B) = 0.7$$

$$(a) T: \text{Normal}(\mu_A, \sigma^2)$$

$$P(T > 3 | A)$$

$$= P\left(\frac{T-2}{3/4} > \frac{3-2}{3/4} \mid A\right)$$

$$= P\left(Z > \frac{4}{3} \mid A\right)$$

$$= 1 - P\left(Z \leq \frac{4}{3} \mid A\right)$$

$$= 1 - P\left(z \leq \frac{4}{3}\right)$$

$$\text{as } z = \frac{T - \mu_A}{\sigma}$$

$$\sim \text{Normal}(0, 1)$$

$$Y \sim N(\mu, \sigma^2)$$

$$Z = \frac{Y - \mu}{\sigma}$$

$$\Rightarrow Z \sim N(0, 1)$$

(b)

$$P(T > 3)$$

$$= P(T > 3 | A) P(A)$$

$$+ P(T > 3 | B) P(B).$$

(c)

$$P(A | T > 3)$$

$$= \frac{P(T > 3 | A) P(A)}{P(T > 3)}$$

etc

Exp

$$U_1 = 2Y - 1$$

$$\therefore Y = \frac{1}{2}(U_1 + 1)$$

$$\therefore f_{U_1}(u) = f_Y\left(\frac{1}{2}(u+1)\right)^{1/2}$$

$$-1 \leq U_1 \leq 1$$

and zero otherwise

$$F_{U_1}(u) = P(U_1 \leq u)$$

$$= P(2Y-1 \leq u)$$

$$= P(Y \leq \frac{1}{2}(u+1))$$

$$= F_Y\left(\frac{1}{2}(u+1)\right)$$

$$U_2 = 1 - 2Y$$

$$F_{U_2}(u) = P(U_2 \leq u)$$

$$= P(1 - 2Y \leq u)$$

$$= P\left(\frac{1}{2}(1 - u) \leq Y\right).$$

$$= 1 - P\left(Y \leq \frac{1}{2}(1 - u)\right)$$

$$= 1 - F_y\left(\frac{1}{2}(1-u)\right)$$

$$(c) U_3 = Y^2$$

$$F_{U_3}(u) = P(U_3 \leq u)$$

$$= P(Y^2 \leq u)$$

$$= P(-\sqrt{u} \leq Y \leq \sqrt{u})$$

$$= F_Y(\sqrt{u}) - F_Y(-\sqrt{u})$$

$$= F_y(\sqrt{v})$$

$$\text{as } F_y(-\sqrt{v}) = 0$$

(U_3 is a non-negative variable)

$$6(a) \quad X = Y^2$$

$$F_X(x) = P(X \leq x)$$

$$= P(Y^2 \leq x)$$

$$= P(-\sqrt{x} \leq Y \leq \sqrt{x})$$

$$= F_Y(\sqrt{x}) - F_Y(-\sqrt{x})$$

$$\therefore f_x(x)$$

$$= \frac{d}{dx} \left\{ F_x(x) \right\}$$

$$= \frac{d}{dx} \left\{ F_y(\sqrt{x}) - F_y(-\sqrt{x}) \right\}.$$

$$= \frac{1}{2} \frac{1}{\sqrt{x}} f_y(\sqrt{x})$$

$$+ \frac{1}{2} \frac{1}{\sqrt{x}} f_y(-\sqrt{x})$$

$$= \frac{1}{\sqrt{x}} f_y(\sqrt{x})$$