

Q1. A psychology experiment studied of different methods of learning names. Three groups of students used different methods to learn the names of the other students in their group. Group 1 used the “simple name game,” Group 2 used the “elaborate name game,” and Group 3 used “pairwise introductions.” The percentage of names recalled (after one year) by each student was recorded as the response variable.

A total sample size of $n = 29$ was obtained. In order to carry out an ANOVA analysis, the following sums of squares quantities were computed:

$$SS = 49.360 \quad SST = 18.822.$$

- (a) Write out the ANOVA table for this analysis. 8 Marks
- (b) Report the result of the ANOVA F-test: state the null hypothesis, and the conclusion of the test. 2 Marks
- (c) Does this analysis utilize balanced replication ? Justify your answer. 2 Marks
- (d) List the three assumptions that are needed for the result of the ANOVA F-test to be valid, and identify one method for checking one of the assumptions. 4 Marks
- (e) Compute the estimate of the residual error variance, σ^2 , in the model used in the analysis. 2 Marks
- (f) Compute the estimate of the **overall** sample variance given by the usual formula

$$\frac{1}{n-1} \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x})^2$$

where n_1, n_2, n_3 are the three Group sample sizes.

2 Marks

Solution:

- (a) ANOVA table: here $k = 3$ is the number of Groups, and $n = 29$, so using the formula sheet we may fill-in as

SOURCE	DF	SS	MS	F
Group	2	18.822	9.411	8.013
ERROR	26	30.538	1.175	
TOTAL	28	49.360		

Figures reported to 3 d.p.

8 Marks

- (b) The null hypothesis is that all Groups have the same mean; here the F-statistic is 8.013, which we compare with the $F_{k-1, n-k} = F_{2, 26}$ distribution. The $\alpha = 0.05$ critical value is therefore

$$F_{\alpha, k-1, n-k} = F_{0.05, 2, 26} = 3.37$$

and we Reject H_0 .

2 Marks

- (c) We have three groups and only 29 data, so this cannot be a balanced. 2 Marks
- (d) We need independent sampling, Normal distribution for the data within groups (i.e. Normal residual errors) and equal variances within the three groups. We cannot check independence, although that is part of the design here; we can check Normality using graphical methods (boxplots, pointplots or histograms), and we can check for equal variances using Levene's Test. 4 Marks

- (e) This estimate is simply MSE , that is 1.175.

2 Marks

- (f) This quantity is simply

$$\frac{SS}{n-1} = \frac{49.360}{28} = 1.763.$$

2 Marks

- Q2. In an analysis of identification capabilities of eyewitnesses, a total of ninety subjects were each shown a video of a simulated crime, and then shortly afterwards shown three sets of 60 photo line-ups. The line-ups contained either 3, 6 or 12 photos each.

Each subject was then instructed to select all the possible suspects they could identify from a specific set of sixty photos (either the first, middle or last set of 60 line-ups). The study was attempting to investigate whether either group size or set number significantly influenced the number of selections made.

An ANOVA table from the analysis of the data is recorded below. The two factors are PHOTOS, which denotes the number of photos shown at a time (either 3, 6 or 12), and SET, which denotes whether questions were focused on the first, middle or last set of 60 line-ups. Some elements of the table have been replaced by dashes.

SOURCE	DF	SS	MS	F
PHOTOS	–	42.075	–	4.035
SET	2	189.867	94.934	–
ERROR	–	443.176	–	
TOTAL	89	675.118		

- (a) Write out the ANOVA table filling in the omitted values. 5 Marks
- (b) Report the conclusions of the ANOVA F-tests 6 Marks
- (c) The ANOVA table is based on an analysis using a model which does not allow for interaction. Suppose that, for this balanced and complete design, when the interaction model is fitted, the sum of squares for interaction, denoted SSI say, takes the value 66.235.
- Carry out the ANOVA F-test for the presence of the interaction, and report the conclusion. 6 Marks
- (d) It is reported that Levene's test yields a p-value of 0.12 for these data. Explain briefly what this test is used for, and what the conclusion from the reported result should be. 3 Marks

In all the tests, you may use the significance level $\alpha = 0.05$.

Solution:

- (a) Here $a = b = 3$, so filling in the entries according to the formula sheet rules yields the following five missing values.

SOURCE	DF	SS	MS	F
PHOTOS	2	42.075	21.037	4.035
SET	2	189.867	94.934	18.208
ERROR	85	443.176	5.214	
TOTAL	89	675.118		

5 Marks

- (b) We compare the two F-values with the $\alpha = 0.05$ upper point of the relevant F-distribution, that is, for PHOTOS and SET

$$F_{0.05, a-1, n-a-b+1} = F_{0.05, b-1, n-a-b+1} = F_{0.05, 2, 85}$$

The table does not tabulate this upper point, but does give $F_{0.05, 2, 60} = 3.15$ and $F_{0.05, 2, 120} = 3.07$, and so both results are significant at the $\alpha = 0.05$ level as the F-statistics exceed these values.

6 Marks

- (c) In this balanced and complete design, the inclusion of an interaction does not influence the factor sums of squares, merely the *SSE*: the *SSI* quantity is **subtracted** from the *SSE* for the original table. The error degrees of freedom becomes $n - ab = 81$. The source degrees of freedom is $(a - 1)(b - 1) = 4$. Therefore, for new test, we have

$$F_I = \frac{MSI}{MSE} = \frac{SSI/(a-1)(b-1)}{SSE/(n-ab)} = \frac{66.235/4}{376.940/81} = \frac{16.559}{4.654} = 3.558.$$

We compare this value against the critical value $F_{0.05, (a-1)(b-1), n-ab} = F_{0.05, 4, 81}$. Again this is not tabulated, but we can use the upper point for the approximation based on the $F_{0.05, 4, 120}$ value to note that

$$3.558 > 2.45$$

so we reject H_0 , and conclude that an interaction should be included. 6 Marks

- (d) Levene's test is a hypothesis test for the equality or homogeneity of variances across the treatment groups defined by the modelled factor level combinations. This p-value indicates that the null hypothesis of equal variances is not rejected.

3 Marks