

Q1. (a) Filling in using the usual rules we have 5 MARKS

- V1 (i) 2 (ii) 105 (iii) 318.509 (iv) 9.007 (v) 17.681
 V2 (i) 94 (ii) 96 (iii) 1077.876 (iv) 9.367 (v) 10.536
 V3 (i) 126 (ii) 129 (iii) 1527.392 (iv) 9.568 (v) 11.212
 V4 (i) 120 (ii) 123 (iii) 1474.669 (iv) 11.783 (v) 1.717
 V5 (i) 3 (ii) 124 (iii) 11.248 (iv) 11.616 (v) 0.323

(b) To complete the test, here are the critical elements 5 MARKS

- V1 F-stat: 17.681. Null dist.: $F_{2,103}$. Crit. val.: $\simeq 3.07$. Result: Reject H_0 .
 V2 F-stat: 10.536. Null dist.: $F_{2,94}$. Crit. val.: $\simeq 3.07$. Result: Reject H_0 .
 V3 F-stat: 11.212. Null dist.: $F_{3,126}$. Crit. val.: $\simeq 2.68$. Result: Reject H_0 .
 V4 F-stat: 1.717. Null dist.: $F_{3,120}$. Crit. val.: $\simeq 2.68$. Result: Do not Reject H_0 .
 V5 F-stat: 0.323. Null dist.: $F_{3,121}$. Crit. val.: $\simeq 2.68$. Result: Do not Reject H_0 .

(c) No difference: balance is not needed for the one factor CRD ANOVA. 2 MARKS

(d) This is the MSE quantity: 4 MARKS

- V1 9.007
 V2 9.367
 V3 9.568
 V4 11.783
 V5 11.616

(e) Levene's test is used to test the equality (or homogeneity) of the treatment group variances. One other check would be for Normality of the within group data, which could be performed using graphical means such as a histogram or boxplot (although this would not be completely definitive, the sample sizes here are quite large) 4 MARKS

- V1 p-value not small, so do not reject the null of equal variances.
 V2 p-value small enough to reject the null of equal variances at $\alpha = 0.05$.
 V3 p-value not small, so do not reject the null of equal variances.
 V4 p-value not small, so do not reject the null of equal variances.
 V5 p-value small enough to reject the null of equal variances at $\alpha = 0.05$.

- Q2. (a) We may read directly from the p-value column and deduce in turn that
 V1 A signif.: $p=0.0000$: B signif.: $p=0.0000$: Interaction not signif.: $p=0.4281$.
 V2 A not signif.: $p=0.3701$: B signif.: $p=0.0173$: Interaction not signif.: $p=0.0586$.
 V3 A not signif.: $p=0.6208$: B signif.: $p=0.0000$: Interaction not signif.: $p=0.0728$.
 V4 A signif.: $p=0.0164$: B signif.: $p=0.0000$: Interaction not signif.: $p=0.1177$.
 V5 A not signif.: $p=0.0920$: B signif.: $p=0.0165$: Interaction not signif.: $p=0.4547$.

6 MARKS

- (b) Here the MS values for the two factors do not change, but the MSE value does change, as the interaction sum of squares and degrees of freedom get subsumed into the error sum of squares and degrees of freedom. That is, we have for the updated version of the F-statistics a new value

6 MARKS

$$F \times \frac{SSE}{n - ab} \times \frac{n - a - b + 1}{SSE + SSI}$$

- V1 Old MSE: 4.399, New MSE: 4.413
 Old F_A : 16.878, New F_A : 16.824, so we still reject H_0
 Old F_B : 22.360, New F_B : 22.289, so we still reject H_0
 V2 Old MSE: 4.361, New MSE: 4.743
 Old F_A : 1.005, New F_A : 0.924, so we still do not reject H_0
 Old F_B : 2.920, New F_B : 2.684, so we still reject H_0 , albeit not as conclusively.
 V3 Old MSE: 4.592, New MSE: 5.031
 Old F_A : 0.595, New F_A : 0.543, so we still do not reject H_0
 Old F_B : 13.115, New F_B : 11.973, so we still reject H_0 .
 V4 Numerical error in table: correct DFs (deduced from MS calcs) should be: 3,2,6,60 and 71. Marks given for calculations using the correct principles.
 V5 Old MSE: 5.331, New MSE: 5.301
 Old F_A : 2.517, New F_A : 2.531, so we still do not reject H_0
 Old F_B : 4.506, New F_B : 4.532, so we still reject H_0 .

- (c) The absence of an interaction (in the model, or deduced from the test result) implies that the effect of the two factors on the outcome is additive, that is, that the effect on the outcome of changing from one level of Factor A to another is the same for each level of Factor B, and vice versa.

3 MARKS

- (d) This is $SS/(n - 1)$ (which is the same whichever model is fitted):

3 MARKS

- V1 9.331
 V2 5.110
 V3 6.488
 V4 5.357 from original table, 7.168 from correct table.
 V5 6.314

- (e) Using correction formula, you need to use

2 MARKS

$$\alpha = 1 - (1 - \alpha_F)^{1/c} \simeq \alpha_F/c$$

where c is the number of tests to be carried out. The number of tests carried out is $c = k(k - 1)/2$ where $k = ab$ is the number of factor level combinations that define individual groups.

General comments:

- a **small** p-value implies we should **reject** the null hypothesis; a small p-value corresponds to a **large** F-statistic value, which leads to a rejection of the null hypothesis as the F_{obs} value exceeds the critical value.
- For 1(d), the **estimate** is written $\hat{\sigma}^2$, the **true value** is written σ^2 ; the MSE is the **estimate**.
- For 2(c): the definition of an interaction is important to understand.
 - An interaction affects the relationship between the response and the factors.
 - It does **not** relate to the relationship between the factors; in a balanced complete factorial design, the experimental units are assigned independently to treatments defined by the factor-level combinations. The factors levels are fixed, so are not referred to as 'independent'.
 - The term 'independent' is reserved in probability to mean something specific; if there is no interaction, it is better to say that the two factors affect the response in an **additive** fashion
 - You can also say that the factors affect the response as **main effects** only