

MATH 204: PRINCIPLES OF STATISTICS 2
WINTER 2026

REGRESSION MODELS: INTERPRETING THE PARAMETERS

The **simple linear** regression model

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

and the **quadratic** regression model

$$Y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$$

might both be used to explain variation in a response variable y as a function of predictor x .

Despite the notation, it is important to recognize that the interpretation of the parameters is **model-specific**. For example, consider the two functions

$$\begin{aligned} m_1(x) &= 2 + 3x \\ m_2(x) &= 2 + 3x + x^2 \end{aligned}$$

that could play the role of mean models in a regression setting: these models might capture the true relationship between the expected value of Y and x .

Both of the models can be written in the regression form

$$m(x) = \mathbb{E}[Y|x] = \beta_0 + \beta_1 x + \beta_2 x^2$$

for specific values of those parameters; in the straight-line model, we have simply that $\beta_2 = 0$. However, when describing the interpretation of β_1 , we need to take note of the assumption about β_2 , and whether it is zero or not. For example:

- in the **straight line** case, it is easy to see that, for each x

$$m_1(x+1) - m_1(x) = (\beta_0 + \beta_1(x+1)) - (\beta_0 + \beta_1 x) = \beta_1.$$

That is, β_1 measures the slope of the line, and quantifies by how much $m_1(x)$ increases as x increases by one unit.

- in the **quadratic** case, we have for each x

$$\begin{aligned} m_2(x+1) - m_2(x) &= (\beta_0 + \beta_1(x+1) + \beta_2(x+1)^2) - (\beta_0 + \beta_1 x + \beta_2 x^2) \\ &= \beta_1 + \beta_2(2x+1) \end{aligned}$$

Now the interpretation of β_1 is not so simple unless $\beta_2 = 0$; it is no longer the case that β_1 measures how much the function increases for a unit increase in x (not least because the amount of increase depends on x itself).

Scaling of predictors: The other important aspect to consider is the **scale** on which parameters should be interpreted. This is tied to the scale of the predictor concerned. For example, suppose we have a measurement of weight, x , made in kilograms (kg), and that some response variable Y is to be measured in financial terms (in dollars), modelled using the simple linear regression model

$$\mathbb{E}[Y|x] = \beta_0 + \beta_1 x.$$

That is, say, if $\beta_0 = 1$ and $\beta_1 = 1.5$, then the response for $x = 2$ (kg) is modelled to have expectation $1 + 1.5 \times 2 = 4$. The expected response is measured in dollars, so β_1 is interpreted on the 'dollar per kilogram', or dollar/kg, scale.

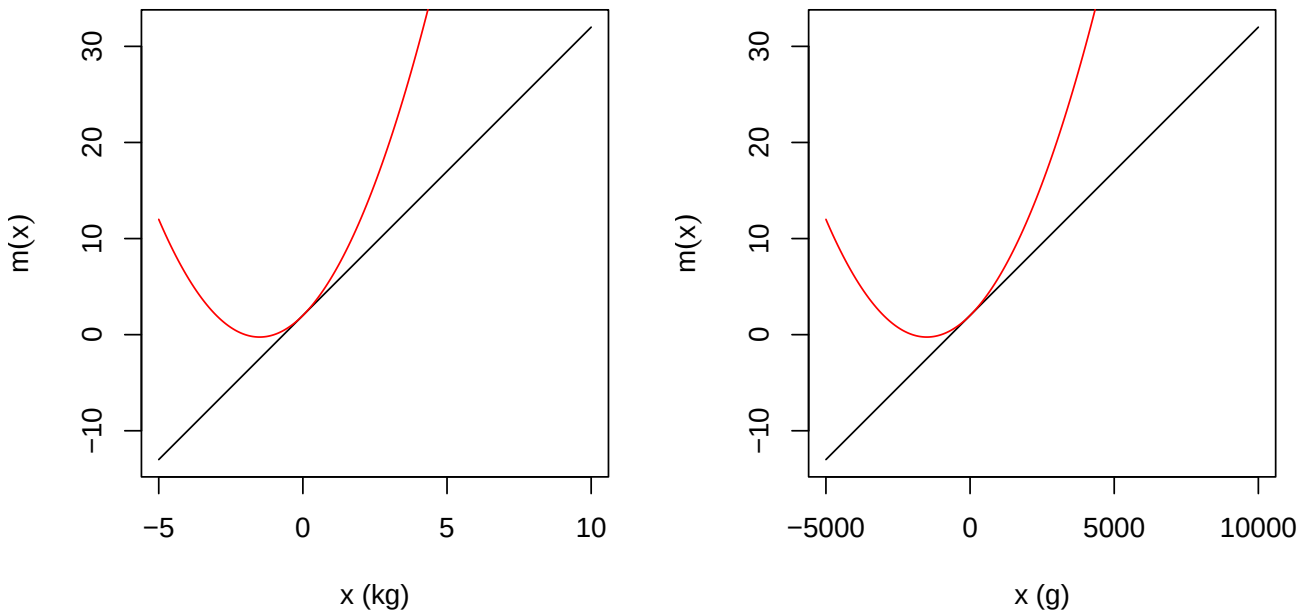
Now suppose that x is instead reported in grams (g). To retain the same response model, this means we must **also** re-scale β_1 , so that the product

$$\beta_1 \times x$$

is left unchanged; if we change scale by 1000 in the move from kg to g, then we must also re-scale β_1 by a factor of 1000. Specifically, for example,

$$1.5 \text{ dollars/kg} = 0.0015 \text{ dollars/g}$$

and the 'new' value of β_1 (on the new x scale) is $1.5/1000 = 0.0015$. For example, if we take the models above, we would have the following equivalent plots. For the straight line, the slope is 1.5 in the first panel, and 0.0015 in the second panel; the vertical axis does not change, but the horizontal axis does.



With this scaling issue in mind for the quadratic model, we also note that x and x^2 may be on dramatically different scales; for example, if we record weights x in kg between 1 and 2, the weight in g varies between 1000 and 2000; for x^2 , the range is from 1 to 4 in kg^2 , but 1000000 to 4000000 in g^2 .