

MATH 204: PRINCIPLES OF STATISTICS 2  
WINTER 2026

HANDOUTS



# MATH 204: PRINCIPLES OF STATISTICS 2

## WINTER 2026

Instructor David A. Stephens (david.stephens@mcgill.ca, Burnside 1225)  
Textbook *Statistics* (13th Edition)  
J. T. McClave and T. Sincich  
Purchasing the textbook is not mandatory; the course will follow the structure of Chapters 10–14, but the course materials will be self-contained.

**Method of Evaluation:** Evaluation for the course will be on the following basis

- **50 % Final Examination**
  - Examination will be in-person and held during the final exam period.
  - Please note that the Instructor has no jurisdiction over the date of the final examination.
- **20 % Midterm Examination**
  - Midterm will be held on **Thursday, 19th February**
  - Midterm examination will be a written exam covering material up to and including Lecture 12 material.
- **15 % Assignments**
  - There will be **two** assignments.
  - Assignments will be a combination of short and long answer questions, submitted electronically.
- **15 % Quizzes**
  - There will be **eight** quizzes, the best **five** will count for the assessment.
  - Quizzes will be open-book time-limited short answer myCourses questions.
  - Quiz topics will be closely related to topics discussed in lectures.

Please see the Course Schedule for the timing of the in-term assessments.

**Note:** The final grade will be determined by the aggregate score

$$70\% \text{ Final Exam} + 15\% \text{ Assignments} + 15\% \text{ Quizzes}$$

if this score exceeds the principal method of evaluation.

The instructor bears sole responsibility for grading issues. Please contact Prof. Stephens (by email at david.stephens@mcgill.ca) if you have any questions concerning grading of any assessed work.

## TARGET COURSE SCHEDULE

| Week |       | Lectures | Topics                                                                                                   | Q  | A  |
|------|-------|----------|----------------------------------------------------------------------------------------------------------|----|----|
| 1    | 05/01 | 1–2      | Outline. Recap of Hypothesis Testing Concepts                                                            |    |    |
| 2    | 12/01 | 3–4      | Designed Experiments. The Analysis of Variance. The Completely Randomized Design. The ANOVA F-test.      |    |    |
| 3    | 19/01 | 5–6      | Completely Randomized Design Examples and Assumptions                                                    | Q1 |    |
| 4    | 26/01 | 7–8      | Blocking and Randomized Block Designs. Replication.                                                      | Q2 |    |
| 5    | 02/02 | 9–10     | Factorial Experiments and multi-factor ANOVA.                                                            | Q3 | A1 |
| 6    | 09/02 | 11–12    | Parameterization. Illegal Models.                                                                        | Q4 |    |
| 7    | 16/02 | 13–14    | Linear regression models. Probabilistic models. Least-squares.                                           |    |    |
| 8    | 23/02 | 15–16    | Model assumptions. Hypothesis testing.                                                                   |    |    |
| 9    | 02/03 | -        | <b>Reading Break</b>                                                                                     | -  | -  |
| 10   | 09/03 | 17–18    | Correlation. $R^2$ statistic. Prediction. ANOVA in regression. Residuals.                                | Q5 |    |
| 11   | 16/03 | 19–20    | Polynomial regression. The linear model. Factor predictors. Multiple linear regression. Model selection. | Q6 |    |
| 12   | 23/03 | 21–22    | ANOVA and F-tests in multiple regression. Unbalanced designs. Pitfalls.                                  | Q7 | A2 |
| 13   | 30/03 | 23–24    | Chi-squared tests for count data. Exact tests. Non-parametric tests.                                     | Q8 |    |
| 14   | 06/04 | 25–26    | Non-parametric tests based on ranks.                                                                     |    |    |

- ‘Q’ refers to Quiz number; ‘A’ refers to Assignment number.
- Assessments will usually be released at the end of the week stated.
- Extra (non-assessed) practice quizzes and other materials may be provided.

### Policy on Late Submission:

- Late submission of the Quizzes will not be possible due to the time-limited and machine-scored nature of the assessment.
- Late submission of the Assignments may be subject to 100% penalty.
- Requests for accommodation due to extenuating circumstances should be directed to Prof. Stephens, [david.stephens@mcgill.ca](mailto:david.stephens@mcgill.ca).

### Policy on the use of AI:

- You may use generative AI tools to assist with coding and with the production of text-based answers. However, anything that you submit for assessment must be **your own work**: for example, it must be the result of **your own use** of the AI tools.
- You are responsible for the work you submit: errors resulting from use of AI tools will be penalized in the usual way.

### Key Academic Dates: <https://www.mcgill.ca/importantdates/key-dates>

- Add/Drop deadline: Tuesday, January 20th
- Winter Reading Break: March 2nd to March 6th inclusive
- Last lecture: Thursday, April 9th
- Classes end: Tuesday, April 14th

### Tutorials & Office Hours:

- There will be two one hour optional tutorials per week; these will begin in Week 2.
- Instructor and Teaching Assistants will also hold weekly Office Hours.

Scheduling details will be announced on myCourses.

### myCourses:

- Official communications will take place via myCourses.
- All course materials will be available via myCourses.

David A. Stephens  
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## UNDERSTANDING THE ANOVA F-STATISTIC

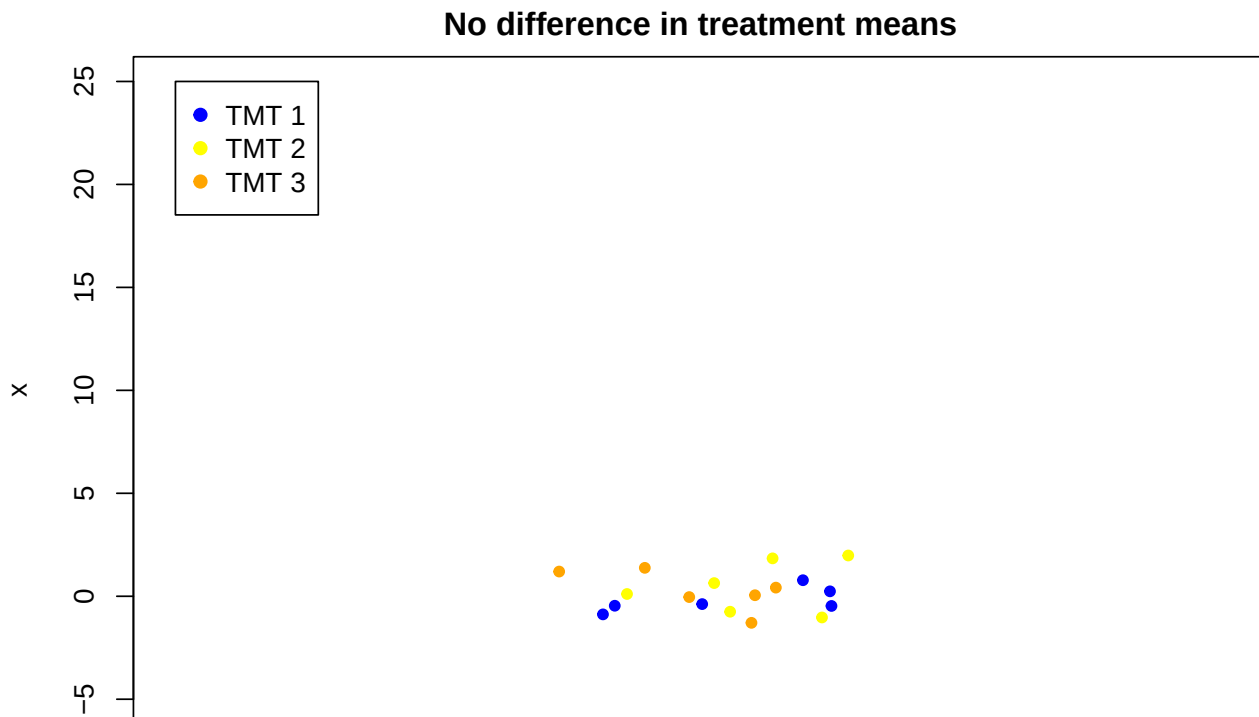
Suppose that we have  $k = 3$  treatment groups in a Completely Randomized Design, with sample sizes  $n_1 = n_2 = n_3 = 6$ . Suppose first that the treatment means are all zero, that is

$$\mu_1 = \mu_2 = \mu_3 = 0$$

and that the treatment group variance parameter  $\sigma^2$  is equal to 1. A typical data set is

|       |       |      |       |      |       |       | $\bar{x}_i$ | $s_i^2$ |
|-------|-------|------|-------|------|-------|-------|-------------|---------|
| TMT 1 | -0.88 | 0.24 | -0.46 | 0.78 | -0.47 | -0.38 | -0.195      | 0.358   |
| TMT 2 | -0.75 | 0.11 | 0.64  | 1.98 | -1.03 | 1.84  | 0.465       | 1.611   |
| TMT 3 | 1.38  | 1.20 | 0.42  | 0.05 | -1.29 | -0.04 | 0.287       | 0.939   |

```
> x<-c(-0.88,0.24,-0.46,0.78,-0.47,-0.38,
+      -0.75,0.11,0.64,1.98,-1.03,1.84,
+      1.38,1.20,0.42,0.05,-1.29,-0.04)
> tmt<-rep(1:3, each=6)
> nvec<-rep(6,3)
> n<-sum(nvec)
> k<-3
> cvec<-c('blue','yellow','orange')
> par(mar=c(1,4,2,2))
> plot(runif(sum(nvec),-3,3),x,col=cvec[tmt],pch=19,cex=0.8,
+      ylim=range(-5,25),xlim=range(-10,10),xaxt='n')
> legend(-10,25,c('TMT 1','TMT 2','TMT 3'),pch=19,col=cvec)
> title('No difference in treatment means')
```



```

> #Summary staistics
> xbar.i<-tapply(x,tmt,mean)
> xvar.i<-tapply(x,tmt,var)
> cbind(1:3,xbar.i,xvar.i)

+           xbar.i    xvar.i
+ 1 1 -0.195000 0.357830
+ 2 2  0.465000 1.610750
+ 3 3  0.286667 0.939187

> xbar<-mean(x)
> xbar

+ [1] 0.185556

```

These data yield  $\bar{x} = 0.185556$ , and

$$s_P^2 = \frac{1}{n-k} \sum_{i=1}^k (n_i - 1) s_i^2 = 0.969.$$

```

> sp.sq<-sum((nvec-1)*xvar.i)/(n-k)
> sp.sq

+ [1] 0.969256

```

For these data, we have using the definitions from lectures

$$SST = \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2 = 1.399 \qquad SSE = \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 = 14.539$$

and

$$SS = \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x})^2 = 15.938$$

so that the equation  $SS = SST + SSE$  holds.

```

> SST<-sum(nvec*(xbar.i-xbar)^2)
> SSE<-sum((x-rep(xbar.i, nvec))^2)
> SS<-sum((x-xbar)^2)
> c(SST, SSE, SS)

+ [1] 1.39881 14.53883 15.93764

```

For the F-statistic, we have

$$F_{obs} = \frac{MST}{MSE} = \frac{SST/(k-1)}{SSE/(n-k)} = \frac{1.399/2}{14.539/15} = 0.722$$

```
> Fobs<-(SST/(k-1))/(SSE/(n-k))
> Fobs

+ [1] 0.72159
```

To complete the test, we compare this with the upper  $\alpha$  probability point of the F-distribution with  $(k-1, n-k) = (2, 15)$  degrees of freedom. With  $\alpha = 0.05$ , from the tables in McClave and Sincich, we see that

$$F_{\alpha,2,15} = 3.68$$

and we **do not reject** the ANOVA F-test null hypothesis

$$H_0 : \mu_1 = \mu_2 = \mu_3.$$

Here, this is the **correct** conclusion, as in fact all the true treatment means are zero. Thus a **small** value of the test statistic  $F$  supports  $H_0$ .

```
> alpha<-0.05
> qf(1-alpha, k-1, n-k)

+ [1] 3.68232
```

Now suppose that, in fact,

$$\mu_1 = 0 \quad \mu_2 = 10 \quad \mu_3 = 20.$$

The equivalent data set to the one above but with the treatment means changed in this way takes the form

|       |       |       |       |       |       |       | $\bar{x}_i$ | $s_i^2$ |
|-------|-------|-------|-------|-------|-------|-------|-------------|---------|
| TMT 1 | -0.88 | 0.24  | -0.46 | 0.78  | -0.47 | -0.38 | -0.195      | 0.358   |
| TMT 2 | 9.25  | 10.11 | 10.64 | 11.98 | 8.97  | 11.84 | 10.465      | 1.611   |
| TMT 3 | 21.38 | 21.20 | 20.42 | 20.05 | 18.71 | 19.96 | 20.287      | 0.939   |

```
> x<-c(-0.88,0.24,-0.46,0.78,-0.47,-0.38,
+      -0.75,0.11,0.64,1.98,-1.03,1.84,
+      1.38,1.20,0.42,0.05,-1.29,-0.04)
> x[7:12]<-x[7:12]+10
> x[13:18]<-x[13:18]+20
> tmt<-rep(1:3, each=6)
> nvec<-rep(6,3)
> n<-sum(nvec)
> k<-3
> cvec<-c('blue','yellow','orange')
> par(mar=c(1,4,2,2))
> plot(runif(sum(nvec),-3,3),x,col=cvec[tmt],pch=19,cex=0.8,
+      ylim=range(-5,25),xlim=range(-10,10),xaxt='n')
> legend(-10,25,c('TMT 1','TMT 2','TMT 3'),pch=19,col=cvec)
> title('Difference in treatment means')
```



so that

$$MST = \frac{1259.199}{2} = 629.600 \quad MSE = \frac{14.539}{15} = 0.969$$

so that

$$F_{obs} = \frac{629.600}{0.969} = 649.570.$$

```
> SST<-sum(nvec*(xbar.i-xbar)^2)
> SSE<-sum((x-rep(xbar.i, nvec))^2)
> SS<-sum((x-xbar)^2)
> c(SST, SSE, SS)

+ [1] 1259.1988 14.5388 1273.7376

> Fobs<-(SST/(k-1))/(SSE/(n-k))
> Fobs

+ [1] 649.57
```

We again compare this with  $F_{\alpha,2,15} = 3.68$  (the critical value,  $C_R$ ), and notice that the  $F$  statistic is **much larger** than this critical value. The test statistic thus lies within the rejection region, and hence we **reject  $H_0$** .

This example illustrates that SST measures the variability **between** means across the treatment groups, whereas SSE measures the variability **within** treatment groups, allowing for the possibility that the treatment means may be different. The quantity SS measures the total amount of variability; in the first example  $SS = SST + SSE$  gives

$$15.938 = 1.399 + 14.539$$

so most of the variability is contributed by SSE, whereas in the second example, we have

$$1273.738 = 1259.199 + 14.539$$

and most of the variability is contributed by SST.

## THE SUMS OF SQUARES DECOMPOSITION

We have seen the fundamental decomposition

$$SS = SST + SSE$$

borne out in numerical calculations. The decomposition is justified as follows:

$$\begin{aligned}
 SS &= \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x})^2 \\
 &= \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i + \bar{x}_i - \bar{x})^2 \\
 &= \sum_{i=1}^k \sum_{j=1}^{n_i} [(x_{ij} - \bar{x}_i)^2 + (\bar{x}_i - \bar{x})^2 + 2(x_{ij} - \bar{x}_i)(\bar{x}_i - \bar{x})] \\
 &= \sum_{i=1}^k \sum_{j=1}^{n_i} (\bar{x}_i - \bar{x})^2 + \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 + 2 \sum_{i=1}^k \sum_{j=1}^{n_i} (\bar{x}_i - \bar{x})(x_{ij} - \bar{x}_i) \\
 &= \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2 + \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 + 0 \\
 &= SST + SSE
 \end{aligned}$$

- Line 2 follows from line 1 by adding  $0 = \bar{x}_i - \bar{x}_i$  inside the bracket.
- Line 3 follows from line 2 by squaring out the bracket

$$(a + b)^2 = a^2 + b^2 + 2ab$$

with  $a = (x_{ij} - \bar{x}_i)$  and  $b = (\bar{x}_i - \bar{x})$ .

- Line 4 follows from line 3 by properties of summation and rearrangement.
- Line 5 follows from line 4 as for the three separate sums, we have for the first term

$$\sum_{i=1}^k \sum_{j=1}^{n_i} (\bar{x}_i - \bar{x})^2 = \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2$$

as the summand  $(\bar{x}_i - \bar{x})^2$  does not depend on  $j$  – it is **constant** in the inner sum, and gets counted  $n_i$  times. For the third term we have

$$\sum_{i=1}^k \sum_{j=1}^{n_i} (\bar{x}_i - \bar{x})(x_{ij} - \bar{x}_i) = \sum_{i=1}^k \left\{ (\bar{x}_i - \bar{x}) \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i) \right\} = 0$$

as the term in the curly bracket is zero for all  $i$  as, by properties of summation,

$$\sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i) = \sum_{j=1}^{n_i} x_{ij} - n_i \bar{x}_i = \sum_{j=1}^{n_i} x_{ij} - \sum_{j=1}^{n_i} x_{ij} = 0.$$

## USING THE F TABLES

Tables in the textbook contain information on the upper  $\alpha$  points for the F-distribution for  $\alpha = 0.1, 0.05, 0.025$  and  $0.01$  respectively, and for different values of the **degrees of freedom** parameters. The values in the body of the table are the numbers  $x$  which solve the equation

$$\Pr[F > x] = \alpha$$

when the statistic  $F$  has a F-distribution with  $\nu_1$  and  $\nu_2$  degrees of freedom, written

$$F \sim F_{\nu_1, \nu_2}$$

where  $\nu_1$  and  $\nu_2$  are whole numbers greater than zero.

The table below is the F table for  $\alpha = 0.05$ , equivalent to the textbook table.

Table 1: Upper  $\alpha = 0.05$  points of  $F_{\nu_1, \nu_2}$  distribution

| $\nu_2$ | $\nu_1$ |        |        |        |        |        |        |        |        |
|---------|---------|--------|--------|--------|--------|--------|--------|--------|--------|
|         | 1       | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      |
| 1       | 161.45  | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| 2       | 18.51   | 19.00  | 19.16  | 19.25  | 19.30  | 19.33  | 19.35  | 19.37  | 19.38  |
| 3       | 10.13   | 9.55   | 9.28   | 9.12   | 9.01   | 8.94   | 8.89   | 8.85   | 8.81   |
| 4       | 7.71    | 6.94   | 6.59   | 6.39   | 6.26   | 6.16   | 6.09   | 6.04   | 6.00   |
| 5       | 6.61    | 5.79   | 5.41   | 5.19   | 5.05   | 4.95   | 4.88   | 4.82   | 4.77   |
| 6       | 5.99    | 5.14   | 4.76   | 4.53   | 4.39   | 4.28   | 4.21   | 4.15   | 4.10   |
| 7       | 5.59    | 4.74   | 4.35   | 4.12   | 3.97   | 3.87   | 3.79   | 3.73   | 3.68   |
| 8       | 5.32    | 4.46   | 4.07   | 3.84   | 3.69   | 3.58   | 3.50   | 3.44   | 3.39   |
| 9       | 5.12    | 4.26   | 3.86   | 3.63   | 3.48   | 3.37   | 3.29   | 3.23   | 3.18   |
| 10      | 4.96    | 4.10   | 3.71   | 3.48   | 3.33   | 3.22   | 3.14   | 3.07   | 3.02   |
| 11      | 4.84    | 3.98   | 3.59   | 3.36   | 3.20   | 3.09   | 3.01   | 2.95   | 2.90   |
| 12      | 4.75    | 3.89   | 3.49   | 3.26   | 3.11   | 3.00   | 2.91   | 2.85   | 2.80   |
| 13      | 4.67    | 3.81   | 3.41   | 3.18   | 3.03   | 2.92   | 2.83   | 2.77   | 2.71   |
| 14      | 4.60    | 3.74   | 3.34   | 3.11   | 2.96   | 2.85   | 2.76   | 2.70   | 2.65   |

From textbook, p 822. Reported to 2 dp.

We read  $\nu_1$  from the **column** and  $\nu_2$  from the **row**. For example,

- if  $\nu_1 = 9$  and  $\nu_2 = 4$ , we know from the table that

$$\Pr[F > 6.00] = 0.05$$

- if  $\nu_1 = 6$  and  $\nu_2 = 3$ , we know from the table that

$$\Pr[F > 8.94] = 0.05$$

- if  $\nu_1 = 2$  and  $\nu_2 = 14$ , we know from the table that

$$\Pr[F > 3.74] = 0.05$$

Table 2: Upper  $\alpha = 0.01$  points of  $F_{\nu_1, \nu_2}$  distribution

| $\nu_2$ | $\nu_1$ |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       |
| 1       | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 |
| 2       | 98.50   | 99.00   | 99.17   | 99.25   | 99.30   | 99.33   | 99.36   | 99.37   | 99.39   |
| 3       | 34.12   | 30.82   | 29.46   | 28.71   | 28.24   | 27.91   | 27.67   | 27.49   | 27.35   |
| 4       | 21.20   | 18.00   | 16.69   | 15.98   | 15.52   | 15.21   | 14.98   | 14.80   | 14.66   |
| 5       | 16.26   | 13.27   | 12.06   | 11.39   | 10.97   | 10.67   | 10.46   | 10.29   | 10.16   |
| 6       | 13.75   | 10.92   | 9.78    | 9.15    | 8.75    | 8.47    | 8.26    | 8.10    | 7.98    |
| 7       | 12.25   | 9.55    | 8.45    | 7.85    | 7.46    | 7.19    | 6.99    | 6.84    | 6.72    |
| 8       | 11.26   | 8.65    | 7.59    | 7.01    | 6.63    | 6.37    | 6.18    | 6.03    | 5.91    |
| 9       | 10.56   | 8.02    | 6.99    | 6.42    | 6.06    | 5.80    | 5.61    | 5.47    | 5.35    |
| 10      | 10.04   | 7.56    | 6.55    | 5.99    | 5.64    | 5.39    | 5.20    | 5.06    | 4.94    |
| 11      | 9.65    | 7.21    | 6.22    | 5.67    | 5.32    | 5.07    | 4.89    | 4.74    | 4.63    |
| 12      | 9.33    | 6.93    | 5.95    | 5.41    | 5.06    | 4.82    | 4.64    | 4.50    | 4.39    |
| 13      | 9.07    | 6.70    | 5.74    | 5.21    | 4.86    | 4.62    | 4.44    | 4.30    | 4.19    |
| 14      | 8.86    | 6.51    | 5.56    | 5.04    | 4.69    | 4.46    | 4.28    | 4.14    | 4.03    |

From textbook, p 826. Reported to 2 dp.

The equivalent table for  $\alpha = 0.01$  is produced above.

Note: The F-distribution is a non-symmetric probability distribution with a specific property that allows the tables to tabulate only the **right-hand tail** of the distribution. If we need to look up the left-hand tail, we can use the fact that if  $F \sim F_{\nu_1, \nu_2}$ , and  $0 < p < 1$

$$\Pr[F > x] = p \quad \implies \quad \Pr[1/F \leq 1/x] = p \quad \implies \quad \Pr[1/F > 1/x] = 1 - p.$$

But it transpires that

$$F \sim F_{\nu_1, \nu_2} \quad \implies \quad \frac{1}{F} \sim F_{\nu_2, \nu_1}.$$

To look up the **left-tail**  $\alpha$  probability point for the  $F \sim F_{\nu_1, \nu_2}$  distribution, we look up the **right-tail**  $1 - \alpha$  probability point for the  $F_{\nu_2, \nu_1}$  distribution, and then take the reciprocal. For example,

- if  $F \sim F_{9,4}$ , we use tables to discover that as  $F_{0.05,4,9} = 3.63$ , it follows that

$$\Pr[F \leq 1/3.63] = \Pr[F \leq 0.28] = 0.05$$

giving the 0.05 (left-tail) probability point of the  $F_{9,4}$  distribution as 0.28.

```
> qf(0.05, 9, 4)
+ [1] 0.275248
> 1/qf(0.95, 4, 9)
+ [1] 0.275248
```

Table 3: Upper  $\alpha = 0.05$  points of  $F_{\nu_1, \nu_2}$  distribution

|    | $\nu_2$ | $\nu_1$ |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
|----|---------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|    |         | 1       | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 12     | 15     | 20     | 24     | 30     | 40     | 60     | 120    | Inf    |
| 14 | 1       | 161.45  | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.95 | 248.01 | 249.05 | 250.10 | 251.14 | 252.20 | 253.25 | 254.31 |
|    | 2       | 18.51   | 19.00  | 19.16  | 19.25  | 19.30  | 19.33  | 19.35  | 19.37  | 19.38  | 19.40  | 19.41  | 19.43  | 19.45  | 19.45  | 19.46  | 19.47  | 19.48  | 19.49  | 19.50  |
|    | 3       | 10.13   | 9.55   | 9.28   | 9.12   | 9.01   | 8.94   | 8.89   | 8.85   | 8.81   | 8.79   | 8.74   | 8.70   | 8.66   | 8.64   | 8.62   | 8.59   | 8.57   | 8.55   | 8.53   |
|    | 4       | 7.71    | 6.94   | 6.59   | 6.39   | 6.26   | 6.16   | 6.09   | 6.04   | 6.00   | 5.96   | 5.91   | 5.86   | 5.80   | 5.77   | 5.75   | 5.72   | 5.69   | 5.66   | 5.63   |
|    | 5       | 6.61    | 5.79   | 5.41   | 5.19   | 5.05   | 4.95   | 4.88   | 4.82   | 4.77   | 4.74   | 4.68   | 4.62   | 4.56   | 4.53   | 4.50   | 4.46   | 4.43   | 4.40   | 4.36   |
|    | 6       | 5.99    | 5.14   | 4.76   | 4.53   | 4.39   | 4.28   | 4.21   | 4.15   | 4.10   | 4.06   | 4.00   | 3.94   | 3.87   | 3.84   | 3.81   | 3.77   | 3.74   | 3.70   | 3.67   |
|    | 7       | 5.59    | 4.74   | 4.35   | 4.12   | 3.97   | 3.87   | 3.79   | 3.73   | 3.68   | 3.64   | 3.57   | 3.51   | 3.44   | 3.41   | 3.38   | 3.34   | 3.30   | 3.27   | 3.23   |
|    | 8       | 5.32    | 4.46   | 4.07   | 3.84   | 3.69   | 3.58   | 3.50   | 3.44   | 3.39   | 3.35   | 3.28   | 3.22   | 3.15   | 3.12   | 3.08   | 3.04   | 3.01   | 2.97   | 2.93   |
|    | 9       | 5.12    | 4.26   | 3.86   | 3.63   | 3.48   | 3.37   | 3.29   | 3.23   | 3.18   | 3.14   | 3.07   | 3.01   | 2.94   | 2.90   | 2.86   | 2.83   | 2.79   | 2.75   | 2.71   |
|    | 10      | 4.96    | 4.10   | 3.71   | 3.48   | 3.33   | 3.22   | 3.14   | 3.07   | 3.02   | 2.98   | 2.91   | 2.85   | 2.77   | 2.74   | 2.70   | 2.66   | 2.62   | 2.58   | 2.54   |
|    | 11      | 4.84    | 3.98   | 3.59   | 3.36   | 3.20   | 3.09   | 3.01   | 2.95   | 2.90   | 2.85   | 2.79   | 2.72   | 2.65   | 2.61   | 2.57   | 2.53   | 2.49   | 2.45   | 2.40   |
|    | 12      | 4.75    | 3.89   | 3.49   | 3.26   | 3.11   | 3.00   | 2.91   | 2.85   | 2.80   | 2.75   | 2.69   | 2.62   | 2.54   | 2.51   | 2.47   | 2.43   | 2.38   | 2.34   | 2.30   |
|    | 13      | 4.67    | 3.81   | 3.41   | 3.18   | 3.03   | 2.92   | 2.83   | 2.77   | 2.71   | 2.67   | 2.60   | 2.53   | 2.46   | 2.42   | 2.38   | 2.34   | 2.30   | 2.25   | 2.21   |
|    | 14      | 4.60    | 3.74   | 3.34   | 3.11   | 2.96   | 2.85   | 2.76   | 2.70   | 2.65   | 2.60   | 2.53   | 2.46   | 2.39   | 2.35   | 2.31   | 2.27   | 2.22   | 2.18   | 2.13   |
|    | 15      | 4.54    | 3.68   | 3.29   | 3.06   | 2.90   | 2.79   | 2.71   | 2.64   | 2.59   | 2.54   | 2.48   | 2.40   | 2.33   | 2.29   | 2.25   | 2.20   | 2.16   | 2.11   | 2.07   |
|    | 16      | 4.49    | 3.63   | 3.24   | 3.01   | 2.85   | 2.74   | 2.66   | 2.59   | 2.54   | 2.49   | 2.42   | 2.35   | 2.28   | 2.24   | 2.19   | 2.15   | 2.11   | 2.06   | 2.01   |
|    | 17      | 4.45    | 3.59   | 3.20   | 2.96   | 2.81   | 2.70   | 2.61   | 2.55   | 2.49   | 2.45   | 2.38   | 2.31   | 2.23   | 2.19   | 2.15   | 2.10   | 2.06   | 2.01   | 1.96   |
|    | 18      | 4.41    | 3.55   | 3.16   | 2.93   | 2.77   | 2.66   | 2.58   | 2.51   | 2.46   | 2.41   | 2.34   | 2.27   | 2.19   | 2.15   | 2.11   | 2.06   | 2.02   | 1.97   | 1.92   |
|    | 19      | 4.38    | 3.52   | 3.13   | 2.90   | 2.74   | 2.63   | 2.54   | 2.48   | 2.42   | 2.38   | 2.31   | 2.23   | 2.16   | 2.11   | 2.07   | 2.03   | 1.98   | 1.93   | 1.88   |
|    | 20      | 4.35    | 3.49   | 3.10   | 2.87   | 2.71   | 2.60   | 2.51   | 2.45   | 2.39   | 2.35   | 2.28   | 2.20   | 2.12   | 2.08   | 2.04   | 1.99   | 1.95   | 1.90   | 1.84   |
|    | 21      | 4.32    | 3.47   | 3.07   | 2.84   | 2.68   | 2.57   | 2.49   | 2.42   | 2.37   | 2.32   | 2.25   | 2.18   | 2.10   | 2.05   | 2.01   | 1.96   | 1.92   | 1.87   | 1.81   |
|    | 22      | 4.30    | 3.44   | 3.05   | 2.82   | 2.66   | 2.55   | 2.46   | 2.40   | 2.34   | 2.30   | 2.23   | 2.15   | 2.07   | 2.03   | 1.98   | 1.94   | 1.89   | 1.84   | 1.78   |
|    | 23      | 4.28    | 3.42   | 3.03   | 2.80   | 2.64   | 2.53   | 2.44   | 2.37   | 2.32   | 2.27   | 2.20   | 2.13   | 2.05   | 2.01   | 1.96   | 1.91   | 1.86   | 1.81   | 1.76   |
|    | 24      | 4.26    | 3.40   | 3.01   | 2.78   | 2.62   | 2.51   | 2.42   | 2.36   | 2.30   | 2.25   | 2.18   | 2.11   | 2.03   | 1.98   | 1.94   | 1.89   | 1.84   | 1.79   | 1.73   |
|    | 25      | 4.24    | 3.39   | 2.99   | 2.76   | 2.60   | 2.49   | 2.40   | 2.34   | 2.28   | 2.24   | 2.16   | 2.09   | 2.01   | 1.96   | 1.92   | 1.87   | 1.82   | 1.77   | 1.71   |
|    | 26      | 4.23    | 3.37   | 2.98   | 2.74   | 2.59   | 2.47   | 2.39   | 2.32   | 2.27   | 2.22   | 2.15   | 2.07   | 1.99   | 1.95   | 1.90   | 1.85   | 1.80   | 1.75   | 1.69   |
|    | 27      | 4.21    | 3.35   | 2.96   | 2.73   | 2.57   | 2.46   | 2.37   | 2.31   | 2.25   | 2.20   | 2.13   | 2.06   | 1.97   | 1.93   | 1.88   | 1.84   | 1.79   | 1.73   | 1.67   |
|    | 28      | 4.20    | 3.34   | 2.95   | 2.71   | 2.56   | 2.45   | 2.36   | 2.29   | 2.24   | 2.19   | 2.12   | 2.04   | 1.96   | 1.91   | 1.87   | 1.82   | 1.77   | 1.71   | 1.65   |
|    | 29      | 4.18    | 3.33   | 2.93   | 2.70   | 2.55   | 2.43   | 2.35   | 2.28   | 2.22   | 2.18   | 2.10   | 2.03   | 1.94   | 1.90   | 1.85   | 1.81   | 1.75   | 1.70   | 1.64   |
|    | 30      | 4.17    | 3.32   | 2.92   | 2.69   | 2.53   | 2.42   | 2.33   | 2.27   | 2.21   | 2.16   | 2.09   | 2.01   | 1.93   | 1.89   | 1.84   | 1.79   | 1.74   | 1.68   | 1.62   |
|    | 40      | 4.08    | 3.23   | 2.84   | 2.61   | 2.45   | 2.34   | 2.25   | 2.18   | 2.12   | 2.08   | 2.00   | 1.92   | 1.84   | 1.79   | 1.74   | 1.69   | 1.64   | 1.58   | 1.51   |
|    | 60      | 4.00    | 3.15   | 2.76   | 2.53   | 2.37   | 2.25   | 2.17   | 2.10   | 2.04   | 1.99   | 1.92   | 1.84   | 1.75   | 1.70   | 1.65   | 1.59   | 1.53   | 1.47   | 1.39   |
|    | 120     | 3.92    | 3.07   | 2.68   | 2.45   | 2.29   | 2.18   | 2.09   | 2.02   | 1.96   | 1.91   | 1.83   | 1.75   | 1.66   | 1.61   | 1.55   | 1.50   | 1.43   | 1.35   | 1.25   |
|    | Inf     | 3.84    | 3.00   | 2.60   | 2.37   | 2.21   | 2.10   | 2.01   | 1.94   | 1.88   | 1.83   | 1.75   | 1.67   | 1.57   | 1.52   | 1.46   | 1.39   | 1.32   | 1.22   | 1.00   |

From textbook, pp 822–823. Reported to 2 dp.

## ONE-WAY ANOVA: WORKED EXAMPLE

A standard model of memory is that the degree to which the subject remembers verbal material is a function of the degree to which it was processed when it was initially presented.

Reference: Craik, F. I. M. and Lockhart, R. S. (1972). Levels of Processing: a framework for memory research. *Journal of Verbal Learning and Verbal Behavior*, 11, 671-684.

Experiment: Fifty subjects aged between 55 and 65 years were randomly assigned to one of five groups which carried out different memory tasks. The five groups included

- The Counting group was asked to read through a list of words and simply count the number of letters in each word.
- The Rhyming group was asked to read each word and think of a word that rhymed with it.
- The Adjective group had to process the words to the extent of giving an adjective that could reasonably be used to modify each word on the list.
- The Imagery group was instructed to try to form vivid images of each word.
- The Intentional group was told to read through the list and to memorize the words for later recall.

After subjects had gone through the list of 27 items three times, they were given a sheet of paper and asked to write down all the words they could remember. The response data were the number of words recalled by each individual in each group, and are presented below:

| Counting | Rhyming | Adjective | Imagery | Intentional |
|----------|---------|-----------|---------|-------------|
| 9        | 7       | 11        | 12      | 10          |
| 8        | 9       | 13        | 11      | 19          |
| 6        | 6       | 8         | 16      | 14          |
| 8        | 6       | 6         | 11      | 5           |
| 10       | 6       | 14        | 9       | 10          |
| 4        | 11      | 11        | 23      | 11          |
| 6        | 6       | 13        | 12      | 14          |
| 5        | 3       | 13        | 10      | 15          |
| 7        | 8       | 10        | 19      | 11          |
| 7        | 7       | 11        | 11      | 11          |

Research question: Does the level of processing required when material is processed affect how much material is remembered ?

```
> load("204DataSets.rda")
> head(Memory)

+           Task Memory
+ 1    Counting      9
+ 2    Rhyming       7
+ 3  Adjective     11
+ 4    Imagery     12
+ 5 Intentional    10
+ 6    Counting      8

> par(mar=c(4, 4, 2, 2))
> boxplot(Memory~Task, data=Memory)
```

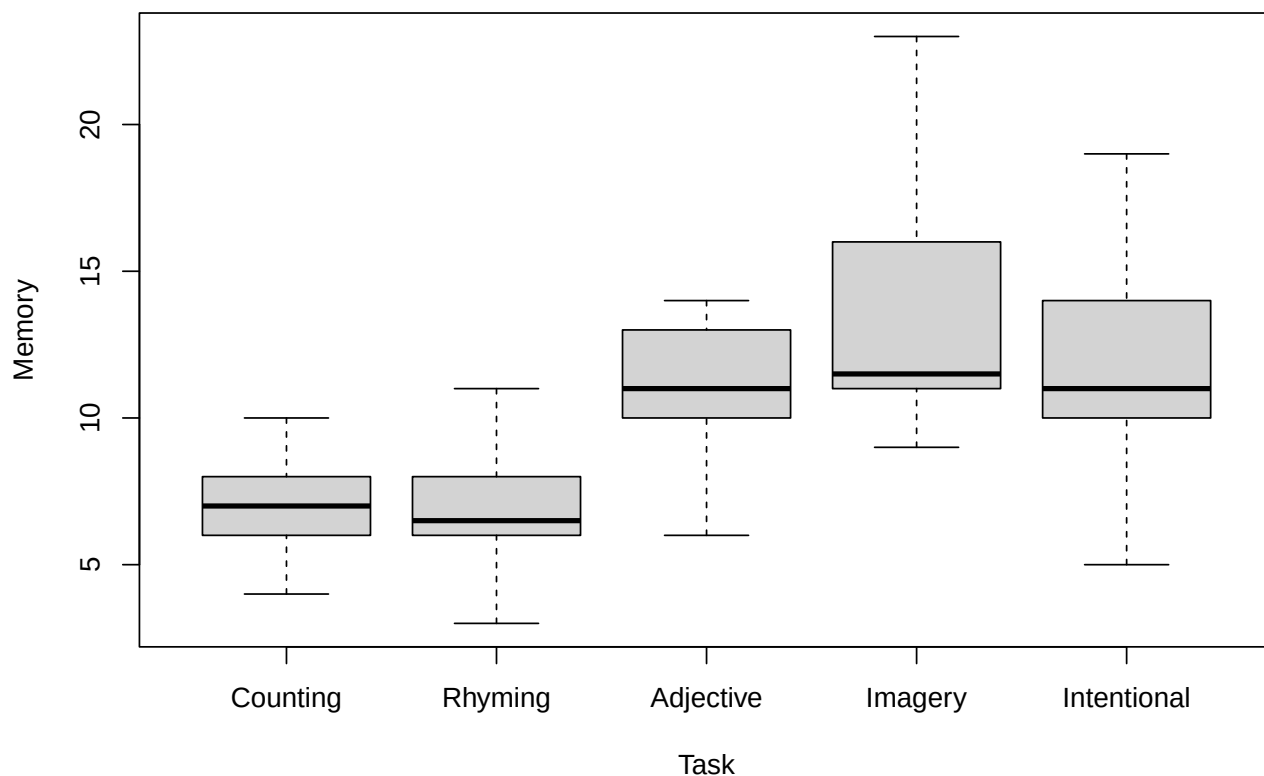


Figure 1: Boxplot of number of errors made by memory task

First we assess the equal variance assumption using Levene's Test from the `car` library.

```
> #Levenes Test
> car::leveneTest(Memory~Task, center='mean', data=Memory)

+ Levene's Test for Homogeneity of Variance (center = "mean")
+      Df F value Pr(>F)
+ group  4    2.529 0.0536 .
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> car::leveneTest(Memory~Task, center='median', data=Memory)

+ Levene's Test for Homogeneity of Variance (center = "median")
+      Df F value Pr(>F)
+ group  4    0.893 0.476
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The null hypothesis of equal group variances is not rejected, using either centering by the mean or centering by the median in the Levene test.

We may proceed with the ANOVA F-test.

```
> print(summary(aov(Memory~Task,data=Memory)), digits = 5)

+           Df Sum Sq Mean Sq F value    Pr(>F)
+ Task         4 351.52   87.880   9.0848 1.815e-05 ***
+ Residuals    45 435.30    9.673
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

**Conclusions:** The  $F$  statistic in this ANOVA is 9.085. This is to be compared with the  $F_{k-1, n-k}$  distribution; here  $k - 1$  is the degrees of freedom for treatment (here for factor `Task`), and  $n - k$  is the residual degrees of freedom. The values are given in the `Df` column as 4 and 45. The result of the test is given by the p-value which is 1.815e-05; hence we **reject** the null hypothesis that there is no difference between treatments at the  $\alpha = 0.05$  significance level.

We can also use the `lm` function to carry out an analysis

```
> summary(lm(Memory~Task,data=Memory))

+
+ Call:
+ lm(formula = Memory ~ Task, data = Memory)
+
+ Residuals:
+    Min       1Q   Median       3Q      Max
+  -7.00  -1.85  -0.45   2.00   9.60
+
+ Coefficients:
+              Estimate Std. Error t value Pr(>|t|)
+ (Intercept)      7.000      0.984    7.12 6.8e-09 ***
+ TaskRhyming     -0.100      1.391   -0.07  0.9430
+ TaskAdjective     4.000      1.391    2.88  0.0061 **
+ TaskImagery       6.400      1.391    4.60 3.4e-05 ***
+ TaskIntentional   5.000      1.391    3.59  0.0008 ***
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
+
+ Residual standard error: 3.11 on 45 degrees of freedom
+ Multiple R-squared:  0.447, Adjusted R-squared:  0.398
+ F-statistic: 9.08 on 4 and 45 DF, p-value: 1.82e-05
```

The result of the ANOVA F-test is given in the final line of output.

## ANOVA FOR THE COMPLETELY RANDOMIZED DESIGN: EXAMPLES

**EXAMPLE** Milk protein level by Diet: data set `Milk`.

```
> par(mar=c(4, 4, 2, 2))
> boxplot(protein~Diet, data=Milk, xlab="Diet", ylab="Protein Level (%)")
```

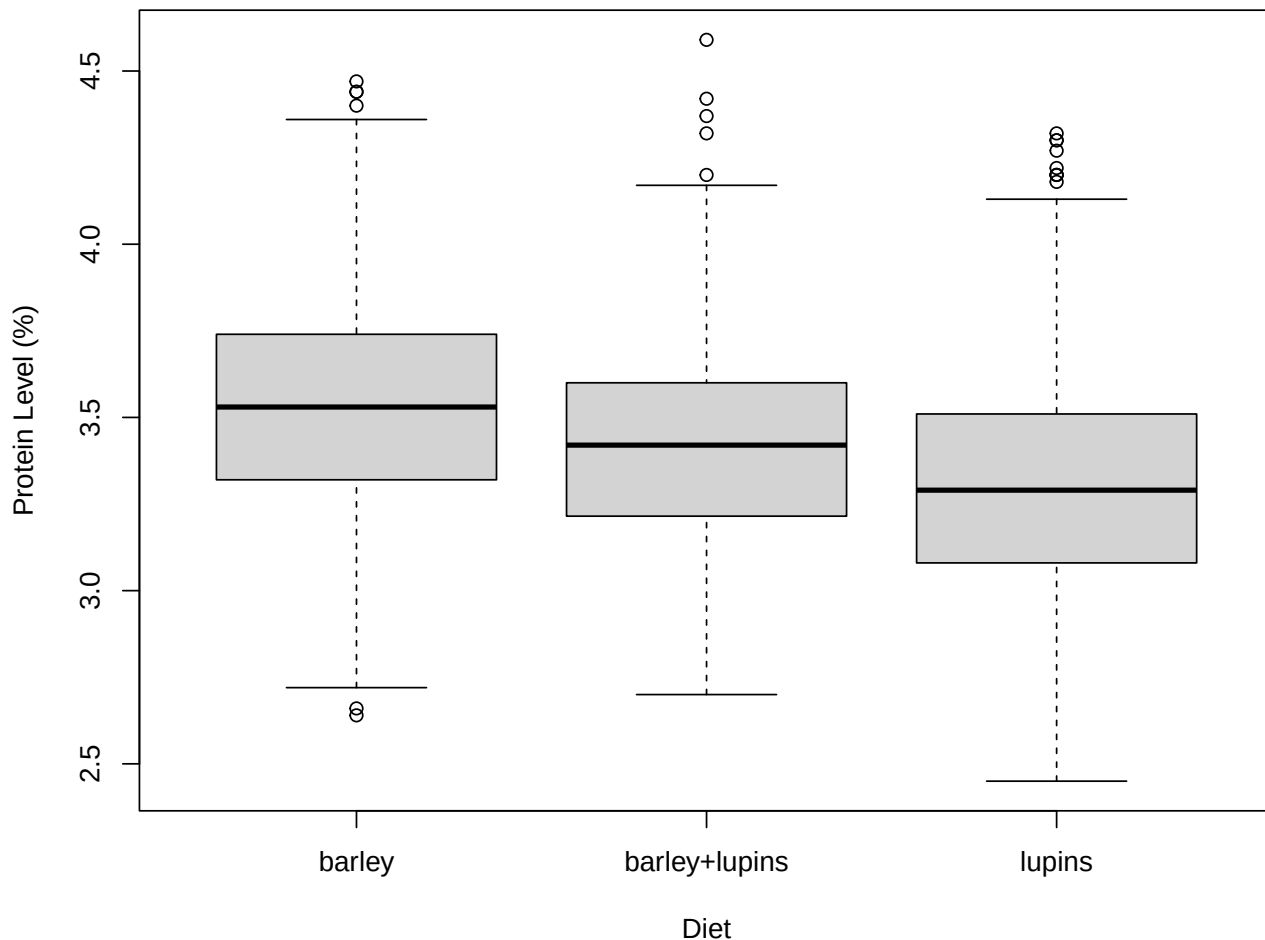


Figure 2: Boxplot of Protein level by Diet

We may write a function to compute basic summary details:

```
> #A summary function
> my.summary<-function(x) {
+
+     my.sum<-data.frame(n=length(x), Mean=mean(x),
+                         StDev=sd(x), StdErr=sd(x)/sqrt(length(x)),
+                         Lower=mean(x)-1.96*sd(x)/sqrt(length(x)),
+                         Upper=mean(x)+1.96*sd(x)/sqrt(length(x)),
+                         Min=min(x), Max=max(x))
+     return(round(my.sum, 5))
+ }
```

Applying this function to the data gives the summaries by Diet.

```
> tapply(Milk$protein, Milk$Diet, my.summary)
+ $barley
+      n      Mean    StDev  StdErr   Lower   Upper   Min   Max
+ 1 425 3.53193 0.31921 0.01548 3.50158 3.56228 2.64 4.47
+
+ $`barley+lupins`
+      n      Mean    StDev  StdErr   Lower   Upper   Min   Max
+ 1 459 3.42969 0.30234 0.01411 3.40204 3.45735 2.7 4.59
+
+ $lupins
+      n      Mean    StDev  StdErr   Lower   Upper   Min   Max
+ 1 453 3.31238 0.33709 0.01584 3.28134 3.34343 2.45 4.32
+
> my.summary(Milk$protein)
+      n      Mean    StDev  StdErr   Lower   Upper   Min   Max
+ 1 1337 3.42245 0.33175 0.00907 3.40466 3.44023 2.45 4.59
```

The Levene test results indicate that the equal variance assumption seems adequate.

```
> car::leveneTest(protein~Diet, center='mean', data=Milk)
+ Levene's Test for Homogeneity of Variance (center = "mean")
+      Df F value Pr(>F)
+ group  2    1.838  0.16
+      1334
+
> car::leveneTest(protein~Diet, center='median', data=Milk)
+ Levene's Test for Homogeneity of Variance (center = "median")
+      Df F value Pr(>F)
+ group  2    1.734  0.177
+      1334
```

We may now compute the sum of squares quantities for the ANOVA F-test using

- aov
- lm
- anova

functions.

```
> #Total
> summary(aov(protein~1, data=Milk))
+      Df Sum Sq Mean Sq F value Pr(>F)
+ Residuals 1336    147    0.11
+
> anova(lm(protein~1, data=Milk))
+ Analysis of Variance Table
+
+ Response: protein
+      Df Sum Sq Mean Sq F value Pr(>F)
+ Residuals 1336    147    0.1101
```

```
> #Fit and Error (Residual)
> summary(aov(protein~Diet,data=Milk))

+           Df Sum Sq Mean Sq F value Pr(>F)
+ Diet           2    10.6      5.3    51.9 <2e-16 ***
+ Residuals    1334   136.4      0.1
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> anova(lm(protein~Diet,data=Milk))

+ Analysis of Variance Table
+
+ Response: protein
+           Df Sum Sq Mean Sq F value Pr(>F)
+ Diet           2   10.61    5.303    51.85 <2e-16 ***
+ Residuals    1334   136.43    0.102
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> summary(lm(protein~Diet,data=Milk))

+
+ Call:
+ lm(formula = protein ~ Diet, data = Milk)
+
+ Residuals:
+      Min       1Q   Median       3Q      Max
+ -0.8919 -0.2197 -0.0124  0.1981  1.1603
+
+ Coefficients:
+             Estimate Std. Error t value Pr(>|t|)
+ (Intercept)      3.5319     0.0155  227.68 < 2e-16 ***
+ Dietbarley+lupins -0.1022     0.0215   -4.75 2.3e-06 ***
+ Dietlupins       -0.2195     0.0216  -10.17 < 2e-16 ***
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
+
+ Residual standard error: 0.32 on 1334 degrees of freedom
+ Multiple R-squared:  0.0721, Adjusted R-squared:  0.0707
+ F-statistic: 51.9 on 2 and 1334 DF, p-value: <2e-16
```

```
> par(mar=c(4,4,1,2))
> gplots::plotmeans(protein~Diet,data=Milk, ylim=range(3.2,3.75))
```

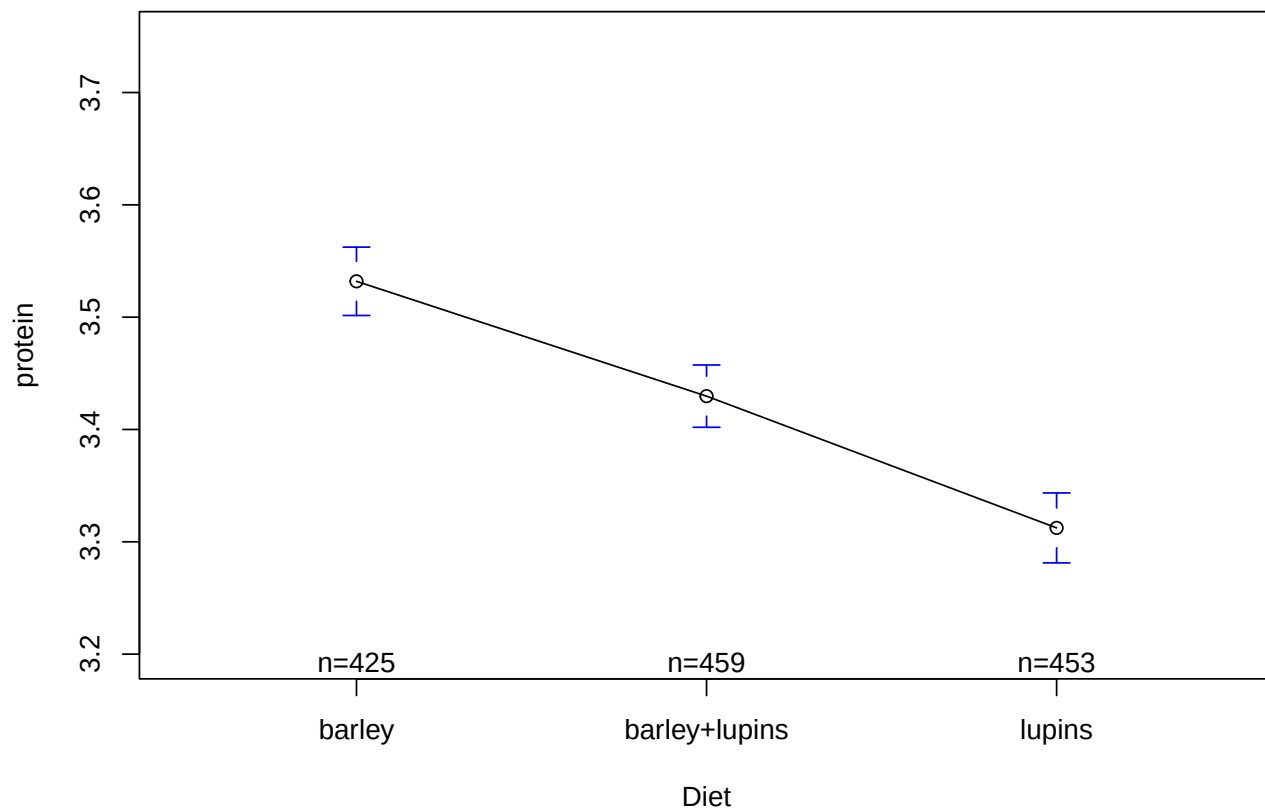


Figure 3: Means plot of Protein level by Diet, with 95 % confidence intervals for means

```
> confint(lm(protein~1+Diet, data=Milk))
+           2.5 %   97.5 %
+ Dietbarley      3.50150 3.56236
+ Dietbarley+lupins 3.40041 3.45898
+ Dietlupins      3.28291 3.34186
```

### EXAMPLE Anxiety level by Dose Level: data set Pull

```
> par(mar=c(4, 4, 2, 2))
> boxplot(Pull~Dose, data=Pull, xlab="Dose", ylab="Pull strength")
```

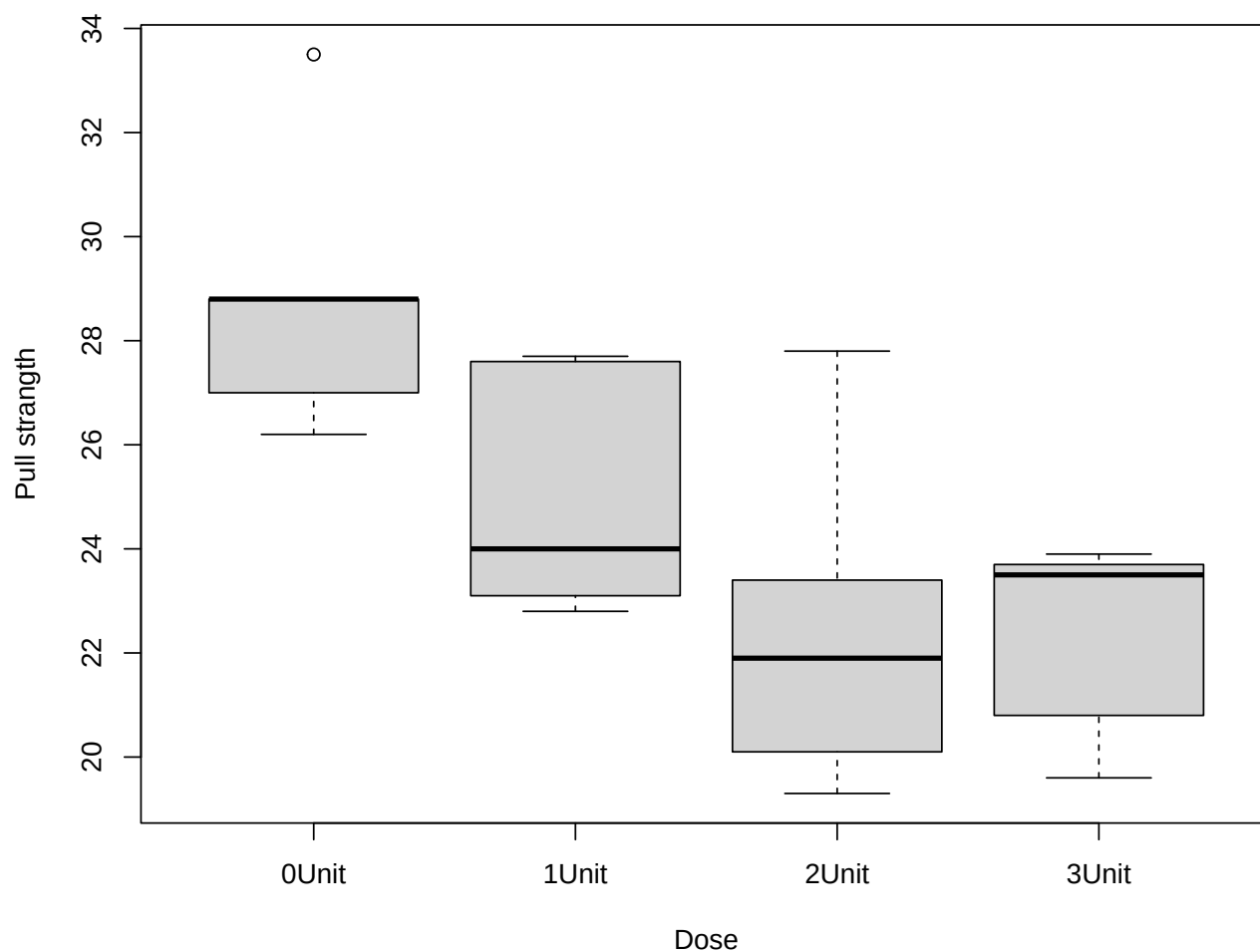


Figure 4: Boxplot of Protein level by Diet

```
> tapply(Pull$Pull, Pull$Dose, my.summary)
+ $`0Unit`
+   n Mean   StDev  StdErr  Lower  Upper  Min  Max
+ 1  5 28.86 2.83161 1.26633 26.378 31.342 26.2 33.5
+
+ $`1Unit`
+   n Mean   StDev  StdErr  Lower  Upper  Min  Max
+ 1  5 25.04 2.42343 1.08379 22.9158 27.1642 22.8 27.7
+
+ $`2Unit`
+   n Mean   StDev  StdErr  Lower  Upper  Min  Max
+ 1  5 22.5 3.36378 1.50433 19.5515 25.4485 19.3 27.8
+
+ $`3Unit`
```

```
+   n Mean   StDev   StdErr   Lower   Upper   Min   Max
+ 1  5 22.3 1.9685 0.88034 20.5745 24.0255 19.6 23.9
```

```
> my.summary(Pull$Pull)
```

```
+   n   Mean   StDev   StdErr   Lower   Upper   Min   Max
+ 1 20 24.675 3.67364 0.82145 23.0649 26.285 19.3 33.5
```

```
> car::leveneTest(Pull~Dose,center='mean',data=Pull)
```

```
+ Levene's Test for Homogeneity of Variance (center = "mean")
+      Df F value Pr(>F)
+ group 3    0.295 0.829
+      16
```

```
> car::leveneTest(Pull~Dose,center='median',data=Pull)
```

```
+ Levene's Test for Homogeneity of Variance (center = "median")
+      Df F value Pr(>F)
+ group 3    0.196 0.898
+      16
```

```
> #Fit and Error (Residual)
```

```
> summary(aov(Pull~Dose,data=Pull))
```

```
+              Df Sum Sq Mean Sq F value Pr(>F)
+ Dose          3    140    46.7    6.42 0.0046 **
+ Residuals    16    116     7.3
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> anova(lm(Pull~Dose,data=Pull))
```

```
+ Analysis of Variance Table
+
+ Response: Pull
+              Df Sum Sq Mean Sq F value  Pr(>F)
+ Dose          3  140.1    46.70    6.423 0.00462 **
+ Residuals    16  116.3     7.27
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> summary(lm(Pull~Dose,data=Pull))
```

```
+
+ Call:
+ lm(formula = Pull ~ Dose, data = Pull)
+
+ Residuals:
+    Min       1Q   Median       3Q      Max
+ -3.20  -2.02  -0.33   1.45   5.30
+
+ Coefficients:
```

```

+           Estimate Std. Error t value Pr(>|t|)
+ (Intercept)    28.86      1.21   23.93  5.9e-14 ***
+ Dose1Unit      -3.82      1.71   -2.24  0.0396 *
+ Dose2Unit      -6.36      1.71   -3.73  0.0018 **
+ Dose3Unit      -6.56      1.71   -3.85  0.0014 **
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
+
+ Residual standard error: 2.7 on 16 degrees of freedom
+ Multiple R-squared:  0.546, Adjusted R-squared:  0.461
+ F-statistic: 6.42 on 3 and 16 DF,  p-value: 0.00462

```

```

> par(mar=c(4,4,0,2))
> gplots::plotmeans(Pull~Dose,data=Pull, ylim=range(15,35))

```

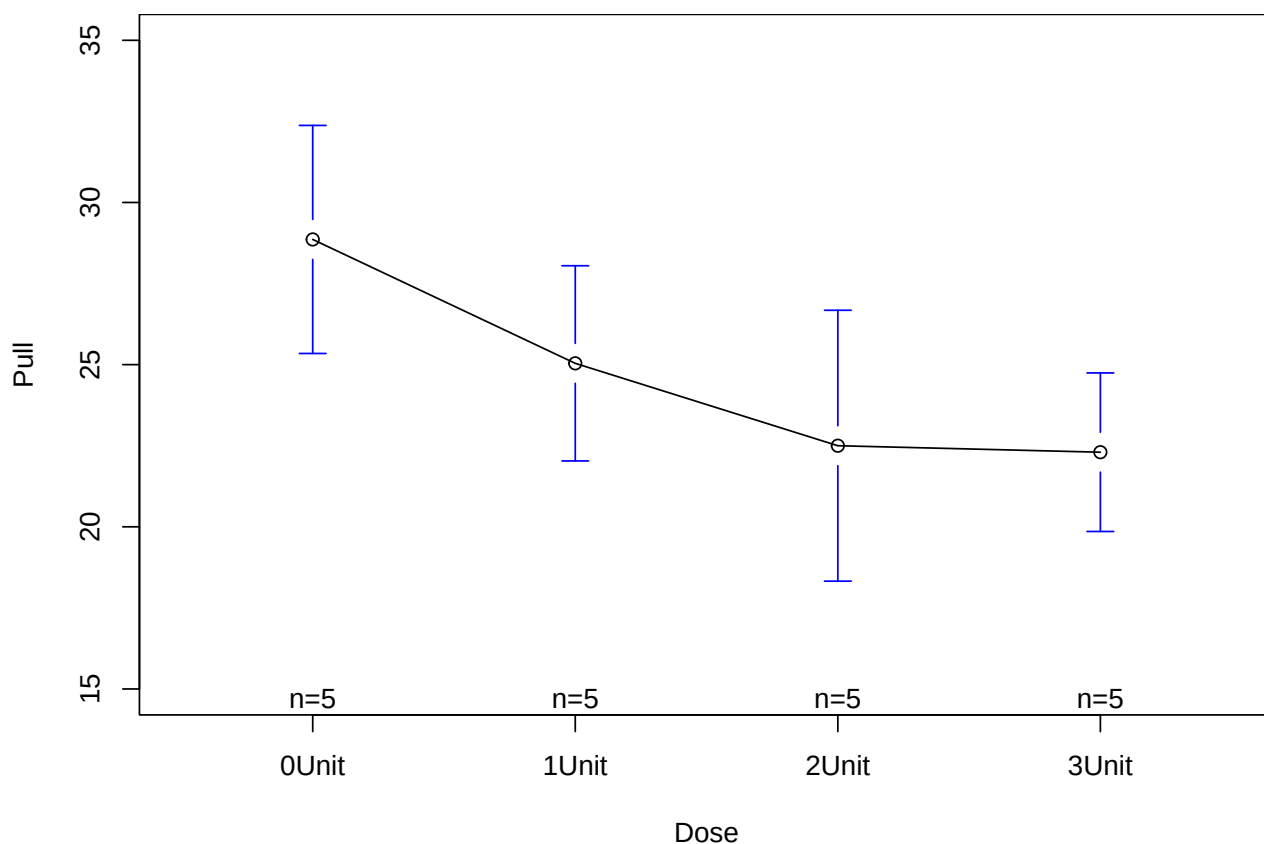


Figure 5: Means plot of pull strength by Dose level, with 95 % confidence intervals for means

```

> #To estimate the Dose-specific means
> summary(lm(Pull~-1+Dose,data=Pull))$coef
+           Estimate Std. Error t value    Pr(>|t|)
+ Dose0Unit    28.86    1.20584  23.9335 5.90944e-14
+ Dose1Unit    25.04    1.20584  20.7656 5.35775e-13
+ Dose2Unit    22.50    1.20584  18.6592 2.78178e-12
+ Dose3Unit    22.30    1.20584  18.4933 3.18983e-12

```

## Post-hoc tests:

```
> summary(aov(Pull~Dose, data=Pull))

+           Df Sum Sq Mean Sq F value Pr(>F)
+ Dose         3    140    46.7    6.42 0.0046 **
+ Residuals   16    116     7.3
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> pairwise.t.test(Pull$Pull, Pull$Dose, p.adj = "none")

+
+ Pairwise comparisons using t tests with pooled SD
+
+ data:  Pull$Pull and Pull$Dose
+
+      0Unit 1Unit 2Unit
+ 1Unit 0.040 -      -
+ 2Unit 0.002 0.156 -
+ 3Unit 0.001 0.128 0.908
+
+ P value adjustment method: none

> pairwise.t.test(Pull$Pull, Pull$Dose, p.adj = "bonferroni")

+
+ Pairwise comparisons using t tests with pooled SD
+
+ data:  Pull$Pull and Pull$Dose
+
+      0Unit 1Unit 2Unit
+ 1Unit 0.238 -      -
+ 2Unit 0.011 0.935 -
+ 3Unit 0.009 0.766 1.000
+
+ P value adjustment method: bonferroni

> TukeyHSD(aov(Pull~Dose, data=Pull))

+ Tukey multiple comparisons of means
+ 95% family-wise confidence level
+
+ Fit: aov(formula = Pull ~ Dose, data = Pull)
+
+ $Dose
+      diff      lwr      upr    p adj
+ 1Unit-0Unit -3.82 -8.69894  1.05894 0.154638
+ 2Unit-0Unit -6.36 -11.23894 -1.48106 0.008884
+ 3Unit-0Unit -6.56 -11.43894 -1.68106 0.006999
+ 2Unit-1Unit -2.54 -7.41894  2.33894 0.466156
+ 3Unit-1Unit -2.74 -7.61894  2.13894 0.402693
+ 3Unit-2Unit -0.20 -5.07894  4.67894 0.999398
```

### EXAMPLE Gut permeability by AIDS diagnosis: data set AIDS

```
> par(mar=c(4, 4, 2, 2))
> boxplot(mann~diag, data=AIDS, xlab="Diagnosis", ylab="Mannitol Level")
```

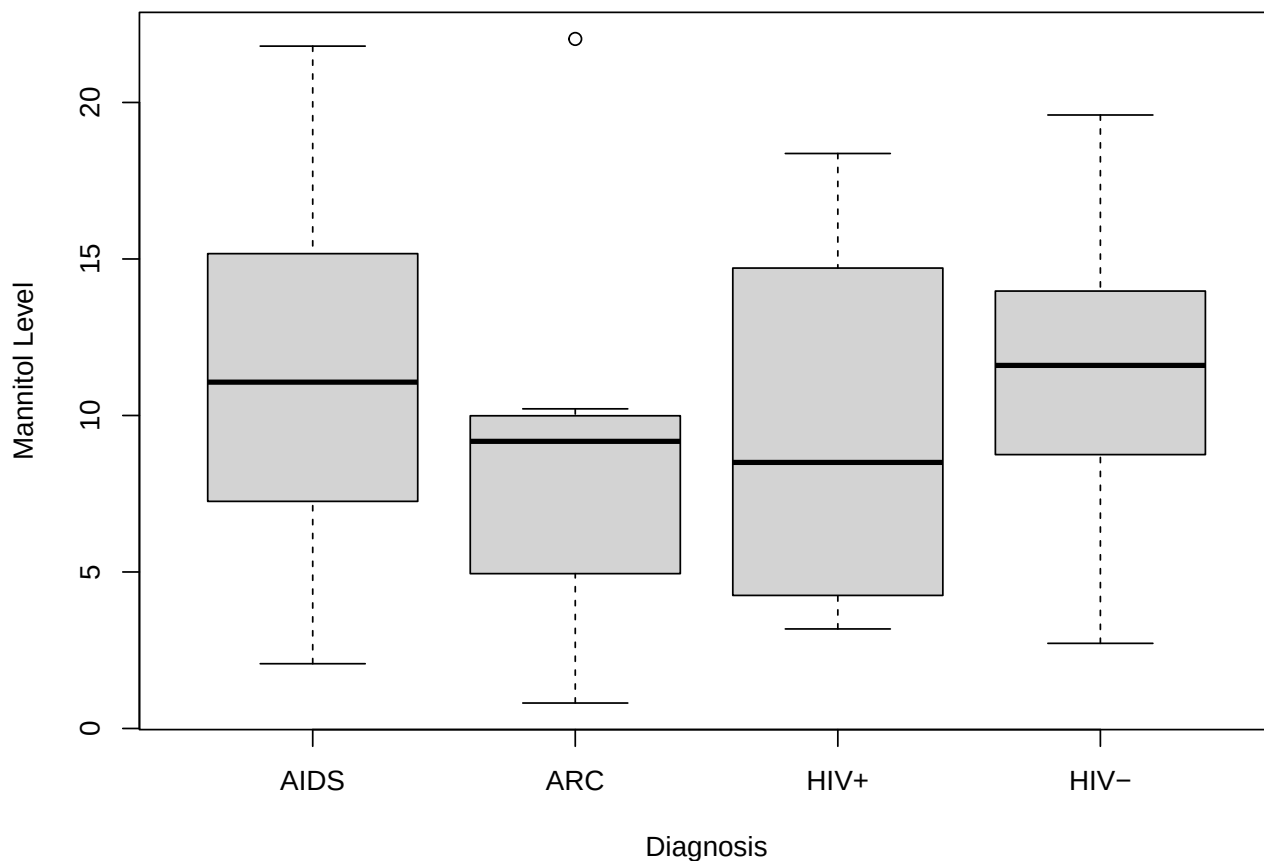


Figure 6: Boxplot of Mannitol level by diagnosis

```
> car::leveneTest(mann~diag, center='mean', data=AIDS)
+ Levene's Test for Homogeneity of Variance (center = "mean")
+   Df F value Pr(>F)
+ group 3    0.866  0.464
+   55

> car::leveneTest(mann~diag, center='median', data=AIDS)
+ Levene's Test for Homogeneity of Variance (center = "median")
+   Df F value Pr(>F)
+ group 3    0.733  0.537
+   55

> summary(aov(mann~diag, data=AIDS))
+           Df Sum Sq Mean Sq F value Pr(>F)
+ diag         3     49    16.3      0.6   0.62
+ Residuals   55    1510    27.4
```

```

> summary(lm(mann~diag,data=AIDS))
+
+ Call:
+ lm(formula = mann ~ diag, data = AIDS)
+
+ Residuals:
+     Min       1Q   Median       3Q      Max
+  -9.262  -3.992   0.169   3.336  13.188
+
+ Coefficients:
+             Estimate Std. Error t value Pr(>|t|)
+ (Intercept)  11.3312     1.0276   11.03  1.5e-15 ***
+ diagARC      -2.4893     2.2312   -1.12    0.27
+ diagHIV+     -1.6208     2.2312   -0.73    0.47
+ diagHIV-      0.0658     1.5814    0.04    0.97
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
+
+ Residual standard error: 5.24 on 55 degrees of freedom
+ Multiple R-squared:  0.0314, Adjusted R-squared:  -0.0214
+ F-statistic: 0.595 on 3 and 55 DF,  p-value: 0.621

```

```

> par(mar=c(4,4,0,2))
> ggplots::plotmeans(mann~diag,data=AIDS,ylim=range(0,18))

```

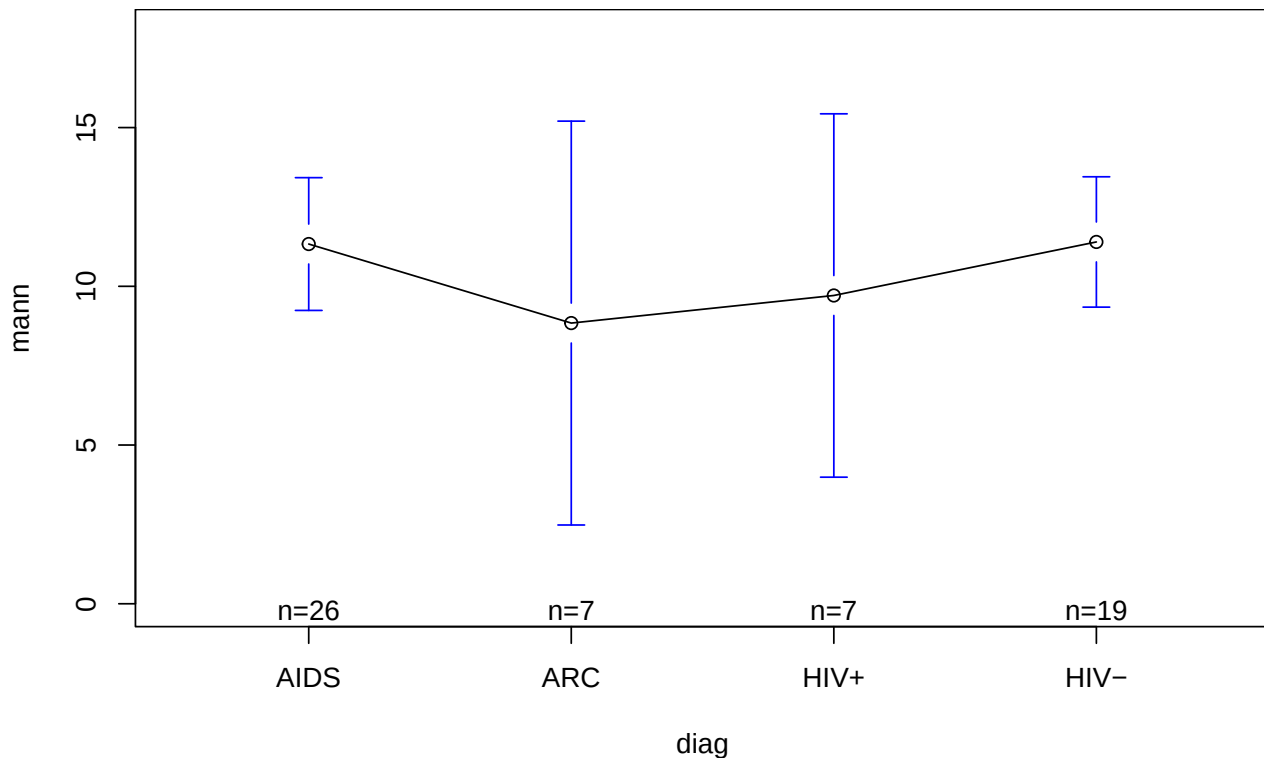


Figure 7: Means plot of gut permeability of mannitol by diagnosis, with 95 % confidence intervals for means

## EXAMPLE Bacteria level in batches of milk: data set MilkBac

```
> car::leveneTest(BacLevel~Batch,center='mean',data=MilkBac)
+ Levene's Test for Homogeneity of Variance (center = "mean")
+      Df F value Pr(>F)
+ group  4    0.384  0.818
+      25

> car::leveneTest(BacLevel~Batch,center='median',data=MilkBac)
+ Levene's Test for Homogeneity of Variance (center = "median")
+      Df F value Pr(>F)
+ group  4    0.243  0.911
+      25

> summary(aov(BacLevel~Batch,data=MilkBac))
+      Df Sum Sq Mean Sq F value   Pr(>F)
+ Batch      4     803   200.8     9.01 0.00012 ***
+ Residuals  25     557    22.3
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> summary(lm(BacLevel~Batch,data=MilkBac))
+
+ Call:
+ lm(formula = BacLevel ~ Batch, data = MilkBac)
+
+ Residuals:
+      Min       1Q   Median       3Q      Max
+  -8.83  -2.54   0.25   2.54   9.17
+
+ Coefficients:
+              Estimate Std. Error t value Pr(>|t|)
+ (Intercept)    23.83      1.93    12.37  3.8e-12 ***
+ Batch2        -10.50      2.73    -3.85  0.00072 ***
+ Batch3        -12.17      2.73    -4.46  0.00015 ***
+ Batch4        -14.67      2.73    -5.38  1.4e-05 ***
+ Batch5         -6.00      2.73    -2.20  0.03716 *
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
+
+ Residual standard error: 4.72 on 25 degrees of freedom
+ Multiple R-squared:  0.59, Adjusted R-squared:  0.525
+ F-statistic: 9.01 on 4 and 25 DF,  p-value: 0.00012

> par(mar=c(4,4,0,2))
> ggplots::plotmeans(BacLevel~Batch,data=MilkBac, ylim=range(0,30))
```

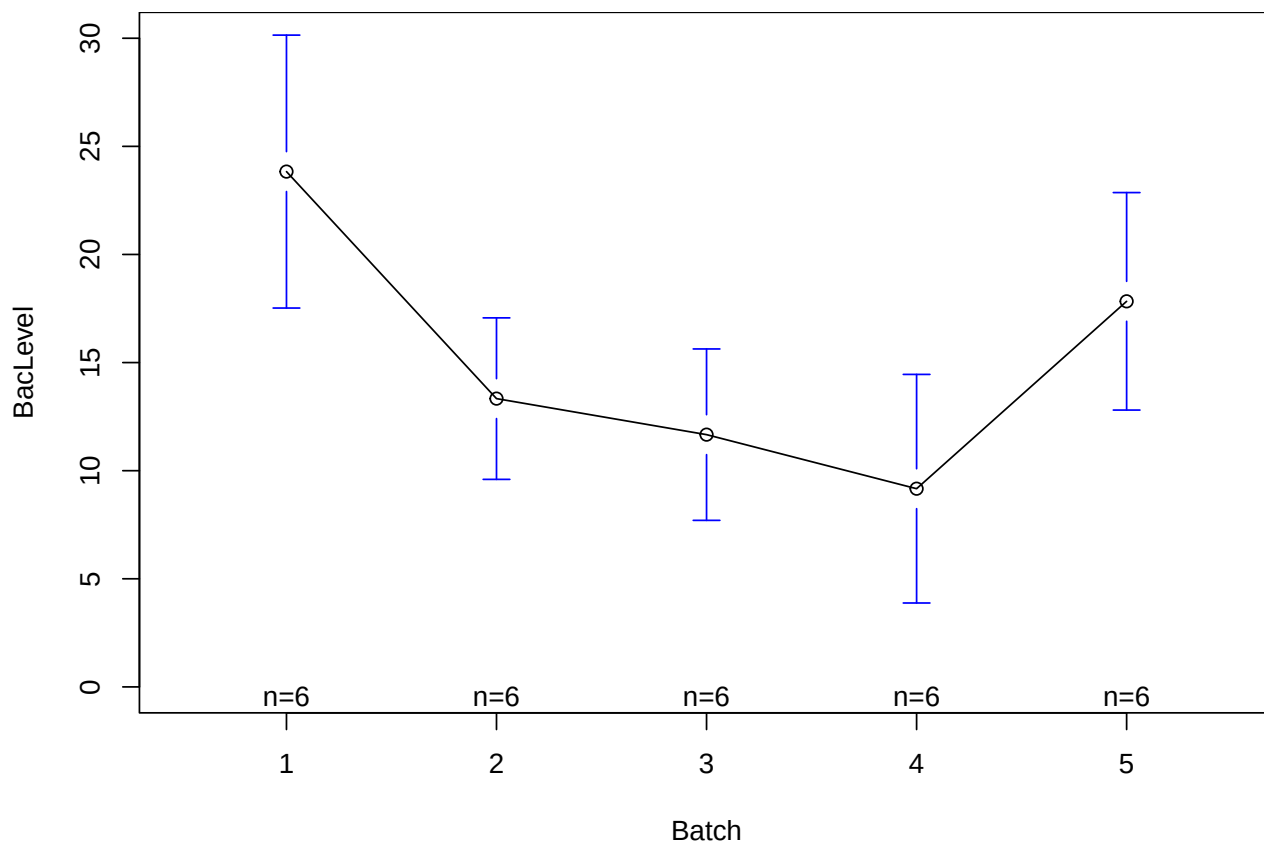


Figure 8: Means plot of bacteria level by batch, with 95 % confidence intervals for means

```
> summary(lm(BacLevel ~ 1 + Batch, data=MilkBac))$coef
```

|          | Estimate | Std. Error | t value  | Pr(> t )    |
|----------|----------|------------|----------|-------------|
| + Batch1 | 23.83333 | 1.92729    | 12.36625 | 3.76038e-12 |
| + Batch2 | 13.33333 | 1.92729    | 6.91818  | 2.98452e-07 |
| + Batch3 | 11.66667 | 1.92729    | 6.05341  | 2.52240e-06 |
| + Batch4 | 9.16667  | 1.92729    | 4.75625  | 7.00794e-05 |
| + Batch5 | 17.83333 | 1.92729    | 9.25306  | 1.49983e-09 |

### EXAMPLE Post-traumatic stress disorder: data set PTSD

Data Source: Foa, E. B., Rothbaum, B. O., Riggs, D. S., & Murdock, T. B. (1991) Treatment of post traumatic stress disorder in rape victims: A comparison between cognitive-behavioral procedures and counseling. *Journal of Consulting and Clinical Psychology*, 59, 715-723.

```
> par(mar=c(4, 4, 2, 2))
> boxplot(PTSD~GROUP, data=PTSD, xlab="Diagnosis", ylab="Mannitol Level")
```

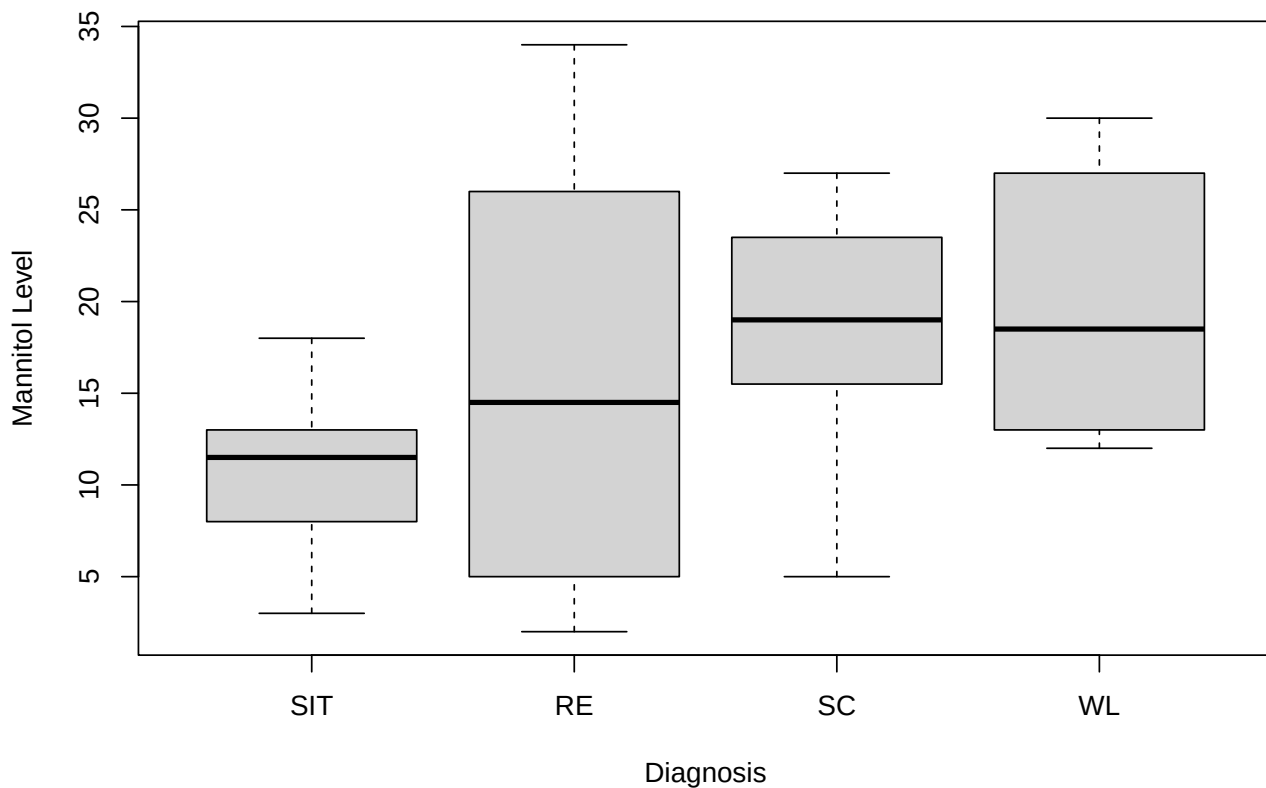


Figure 9: Boxplot of PTSD score by therapy

```
> car::leveneTest(PTSD~GROUP, center='mean', data=PTSD)
+ Levene's Test for Homogeneity of Variance (center = "mean")
+      Df F value  Pr(>F)
+ group 3    6.633 0.000934 ***
+    41
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> car::leveneTest(PTSD~GROUP, center='median', data=PTSD)
+ Levene's Test for Homogeneity of Variance (center = "median")
+      Df F value  Pr(>F)
+ group 3    6.171 0.00147 **
+    41
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```

> summary(aov(PTSD~GROUP,data=PTSD))

+           Df Sum Sq Mean Sq F value Pr(>F)
+ GROUP          3      508   169.3    3.05  0.039 *
+ Residuals    41     2279    55.6
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> summary(lm(PTSD~GROUP,data=PTSD))

+
+ Call:
+ lm(formula = PTSD ~ GROUP, data = PTSD)
+
+ Residuals:
+     Min       1Q   Median       3Q      Max
+ -13.40  -4.40   0.50   4.93  18.60
+
+ Coefficients:
+             Estimate Std. Error t value Pr(>|t|)
+ (Intercept)    11.07      1.99     5.56 1.8e-06 ***
+ GROUPRE         4.33      3.09     1.40  0.1683
+ GROUpsc         7.02      3.00     2.34  0.0244 *
+ GROUPL          8.43      3.09     2.73  0.0093 **
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
+
+ Residual standard error: 7.46 on 41 degrees of freedom
+ Multiple R-squared:  0.182, Adjusted R-squared:  0.122
+ F-statistic: 3.05 on 3 and 41 DF,  p-value: 0.0394

> par(mar=c(4,4,1,2))
> ggplot2::plotmeans(PTSD~GROUP,data=PTSD, ylim=range(0,30))

```

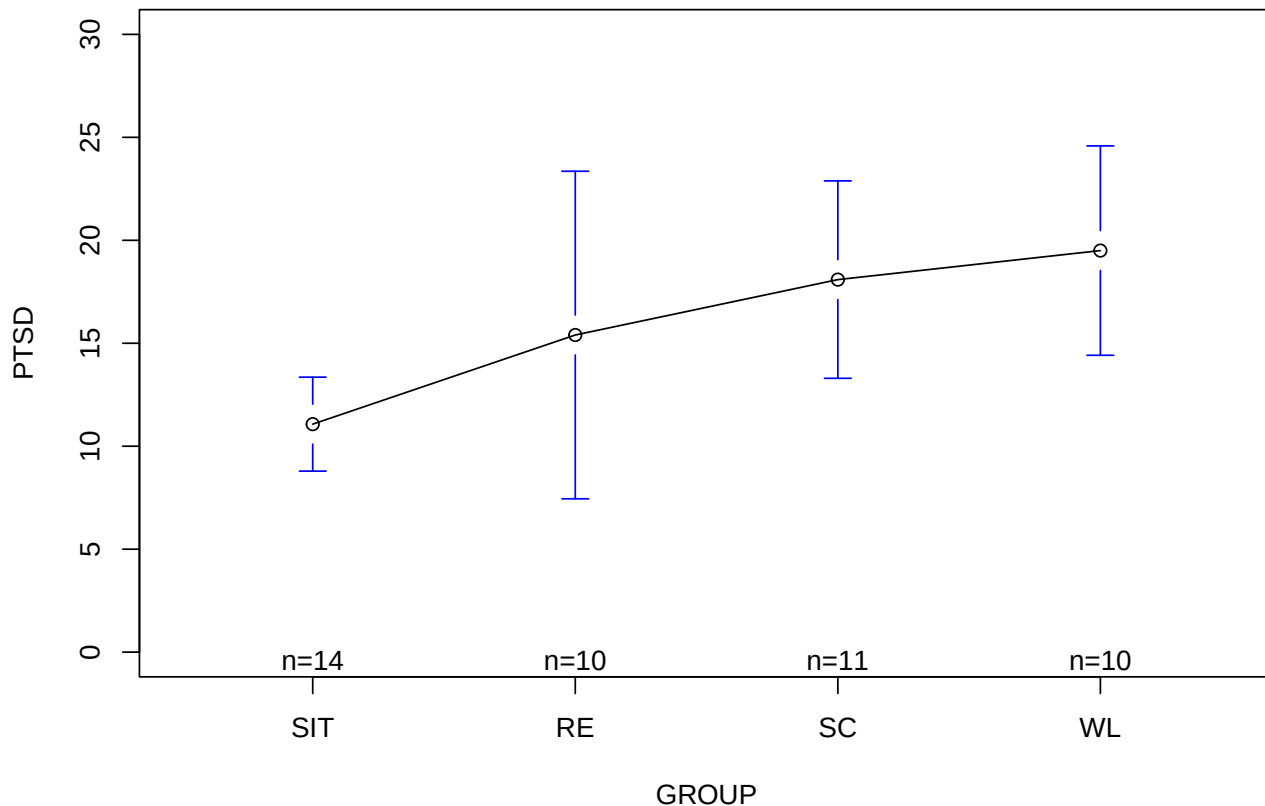


Figure 10: Means plot of PTSD score by therapy, with 95 % confidence intervals for means

```
> summary(lm(PTSD~1+GROUP, data=PTSD))$coef
```

```
+      Estimate Std. Error t value    Pr(>|t|)
+ GROUPSIT    11.0714    1.99247  5.55664 1.84459e-06
+ GROUPRE     15.4000    2.35752  6.53229 7.57159e-08
+ GROUPSC     18.0909    2.24781  8.04825 5.69479e-10
+ GROUPWL     19.5000    2.35752  8.27141 2.81770e-10
```

```
> summary(aov(PTSD~GROUP, data=PTSD))
```

```
+      Df Sum Sq Mean Sq F value Pr(>F)
+ GROUP      3      508    169.3      3.05  0.039 *
+ Residuals  41     2279     55.6
+ ---
+ Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> pairwise.t.test(PTSD$PTSD, PTSD$GROUP, p.adj = "none")
```

```
+
+ Pairwise comparisons using t tests with pooled SD
+
+ data: PTSD$PTSD and PTSD$GROUP
+
+      SIT    RE    SC
+ RE 0.168 -      -
+ SC 0.024 0.414 -
```

```

+ WL 0.009 0.226 0.668
+
+ P value adjustment method: none

> pairwise.t.test(PTSD$PTSD, PTSD$GROUP, p.adj = "bonferroni")

+
+ Pairwise comparisons using t tests with pooled SD
+
+ data: PTSD$PTSD and PTSD$GROUP
+
+ SIT RE SC
+ RE 1.00 - -
+ SC 0.15 1.00 -
+ WL 0.06 1.00 1.00
+
+ P value adjustment method: bonferroni

```

```

> TukeyHSD(aov(PTSD~GROUP, data=PTSD))

+ Tukey multiple comparisons of means
+ 95% family-wise confidence level
+
+ Fit: aov(formula = PTSD ~ GROUP, data = PTSD)
+
+ $GROUP
+ diff lwr upr p adj
+ RE-SIT 4.32857 -3.936483 12.5936 0.505162
+ SC-SIT 7.01948 -1.023440 15.0624 0.106248
+ WL-SIT 8.42857 0.163517 16.6936 0.044145
+ SC-RE 2.69091 -6.031118 11.4129 0.841777
+ WL-RE 4.10000 -4.827279 13.0273 0.611888
+ WL-SC 1.40909 -7.312936 10.1311 0.972524

```