

MATH 204: PRINCIPLES OF STATISTICS 2
WINTER 2026

EXERCISES ON HYPOTHESIS TESTING IN TWO SAMPLE PROBLEMS

In the following two-sample problems, with samples of size n_A and n_B , we wish to test the null hypothesis

$$H_0 : \mu_A = \mu_B$$

against the alternative, using the significance level α , as given in the question. The summary statistics needed are the sample mean and sample variance for the two samples,

$$(\bar{x}_A, s_A^2) \quad \text{and} \quad (\bar{x}_B, s_B^2).$$

We will test under the assumptions of

- Normality for both samples
- Independence of the samples
- Equal variances, $\sigma_A^2 = \sigma_B^2 = \sigma^2$, say.

We will use the test statistic, T , where the observed value of the test statistic is computed as

$$T_{obs} = \frac{\bar{x}_A - \bar{x}_B}{s_P \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}}$$

where s_P^2 is the pooled estimate of σ^2 ,

$$s_P^2 = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}.$$

If the null hypothesis is true, we have that the distribution of T is

$$T \sim t_{n_A + n_B - 2}$$

that is the t-distribution with $n_A + n_B - 2$ degrees of freedom.

- If both n_A and n_B are greater than 15, we may approximate this distribution using the standard Normal distribution if necessary: the Normal distribution is the limiting case of t-distribution where the degrees of freedom is infinite.
- A suitable table of the t-distribution can be found here:

Math204-Winter2026-tTable.pdf

Problem	n_A	$\bar{x}_A$	s_A	n_B	$\bar{x}_B$	s_B	α	H_a
1	6	-4.54	0.86	19	0.55	1.21	0.05	$\mu_A > \mu_B$
2	7	1.43	4.42	12	1.13	3.61	0.05	$\mu_A < \mu_B$
3	7	-2.12	1.90	20	1.98	1.75	0.10	$\mu_A < \mu_B$
4	20	1.09	2.07	18	1.90	1.7	0.01	$\mu_A < \mu_B$
5	19	-0.13	1.06	13	-3.33	0.85	0.05	$\mu_A < \mu_B$
6	20	-5.59	1.28	7	-1.31	1.53	0.01	$\mu_A > \mu_B$
7	19	-0.77	2.14	20	-3.11	1.48	0.05	$\mu_A \neq \mu_B$
8	10	-2.38	1.97	8	-3.59	1.82	0.05	$\mu_A > \mu_B$
9	11	0.45	2.64	20	2.04	1.92	0.05	$\mu_A > \mu_B$
10	12	3.31	2.03	8	3.31	1.14	0.10	$\mu_A > \mu_B$
11	10	4.01	1.22	13	-2.23	1.34	0.05	$\mu_A < \mu_B$
12	10	-3.81	1.20	14	-2.16	0.63	0.10	$\mu_A > \mu_B$
13	9	3.47	1.89	16	1.41	2.77	0.05	$\mu_A < \mu_B$
14	18	-0.03	1.80	9	-3.70	2.13	0.10	$\mu_A > \mu_B$
15	20	3.25	4.18	11	-1.83	2.9	0.05	$\mu_A \neq \mu_B$
16	12	2.29	1.06	18	0.66	1.08	0.10	$\mu_A < \mu_B$
17	19	3.78	1.04	19	1.88	1.09	0.01	$\mu_A \neq \mu_B$
18	17	-3.04	3.14	5	1.57	2.14	0.05	$\mu_A \neq \mu_B$
19	8	-3.23	2.13	5	-5.49	2.1	0.01	$\mu_A < \mu_B$
20	5	-0.12	1.62	14	3.19	1.4	0.05	$\mu_A < \mu_B$
21	15	0.88	1.26	17	1.33	1.3	0.05	$\mu_A \neq \mu_B$
22	18	2.44	1.82	10	-4.66	1.96	0.05	$\mu_A < \mu_B$
23	9	1.68	1.33	19	3.23	1.33	0.05	$\mu_A > \mu_B$
24	7	0.72	0.90	9	0.37	1.41	0.10	$\mu_A < \mu_B$
25	6	5.30	1.97	5	-3.09	1.3	0.10	$\mu_A < \mu_B$

Problem	n_A	$\bar{x}_A$	s_A	n_B	$\bar{x}_B$	s_B	α	H_a
26	6	-0.18	0.88	12	-2.70	1.41	0.05	$\mu_A > \mu_B$
27	12	5.51	2.60	13	-1.19	2.24	0.10	$\mu_A > \mu_B$
28	14	1.39	1.17	11	4.22	1.7	0.10	$\mu_A > \mu_B$
29	18	-3.71	1.71	11	-3.61	2.11	0.01	$\mu_A \neq \mu_B$
30	14	-4.62	1.44	20	0.93	1.49	0.10	$\mu_A > \mu_B$
31	10	1.44	2.50	15	-1.71	1.45	0.05	$\mu_A > \mu_B$
32	10	-0.42	1.19	17	1.35	0.8	0.10	$\mu_A \neq \mu_B$
33	12	2.76	1.31	12	-0.60	1.94	0.10	$\mu_A < \mu_B$
34	10	0.33	1.80	14	-0.18	2.4	0.10	$\mu_A < \mu_B$
35	13	4.93	2.76	19	-1.74	2.98	0.10	$\mu_A < \mu_B$
36	12	-0.12	1.81	20	5.50	1.25	0.01	$\mu_A > \mu_B$
37	20	-4.37	1.68	16	1.56	1.49	0.10	$\mu_A > \mu_B$
38	11	2.71	0.59	10	2.95	0.74	0.10	$\mu_A > \mu_B$
39	10	-0.59	0.84	20	1.71	0.67	0.01	$\mu_A > \mu_B$
40	14	-1.40	2.82	16	-3.96	2.52	0.10	$\mu_A > \mu_B$
41	18	4.26	0.78	5	-0.86	0.8	0.05	$\mu_A \neq \mu_B$
42	10	-1.78	1.66	20	1.52	2.27	0.10	$\mu_A \neq \mu_B$
43	15	0.10	1.93	6	2.24	2.18	0.10	$\mu_A < \mu_B$
44	20	-3.39	2.08	16	3.93	2.49	0.05	$\mu_A \neq \mu_B$
45	12	-4.45	1.29	13	-2.65	2.16	0.10	$\mu_A > \mu_B$
46	6	-1.93	1.98	8	0.92	1.71	0.10	$\mu_A < \mu_B$
47	17	-0.01	1.73	15	0.27	2.39	0.10	$\mu_A \neq \mu_B$
48	8	3.38	2.21	20	3.73	1.64	0.05	$\mu_A < \mu_B$
49	6	-1.96	3.29	6	1.89	2.22	0.05	$\mu_A < \mu_B$
50	8	2.86	2.28	9	4.67	2.81	0.01	$\mu_A \neq \mu_B$

Solutions: The column labelled 'Crit.' is the critical value, that is, the boundary of the Rejection Region; in a two-tailed test, the upper critical value is given only. The results that Reject H_0 are indicated.

	n_A	$\bar{x}_A$	s_A	n_B	$\bar{x}_B$	s_B	α	H_a	T_{obs}	Crit.	Result
1	6	-4.54	0.86	19	0.55	1.21	0.05	$\mu_A > \mu_B$	-9.509	1.714	
2	7	1.43	4.42	12	1.13	3.61	0.05	$\mu_A < \mu_B$	0.161	-1.740	
3	7	-2.12	1.90	20	1.98	1.75	0.10	$\mu_A < \mu_B$	-5.224	-1.316	Reject
4	20	1.09	2.07	18	1.90	1.7	0.01	$\mu_A < \mu_B$	-1.309	-2.434	
5	19	-0.13	1.06	13	-3.33	0.85	0.05	$\mu_A < \mu_B$	9.059	-1.697	
6	20	-5.59	1.28	7	-1.31	1.53	0.01	$\mu_A > \mu_B$	-7.250	2.485	
7	19	-0.77	2.14	20	-3.11	1.48	0.05	$\mu_A \neq \mu_B$	3.989	2.026	Reject
8	10	-2.38	1.97	8	-3.59	1.82	0.05	$\mu_A > \mu_B$	1.338	1.746	
9	11	0.45	2.64	20	2.04	1.92	0.05	$\mu_A > \mu_B$	-1.930	1.699	
10	12	3.31	2.03	8	3.31	1.14	0.10	$\mu_A > \mu_B$	0.000	1.330	
11	10	4.01	1.22	13	-2.23	1.34	0.05	$\mu_A < \mu_B$	11.501	-1.721	
12	10	-3.81	1.20	14	-2.16	0.63	0.10	$\mu_A > \mu_B$	-4.391	1.321	
13	9	3.47	1.89	16	1.41	2.77	0.05	$\mu_A < \mu_B$	1.978	-1.714	
14	18	-0.03	1.80	9	-3.70	2.13	0.10	$\mu_A > \mu_B$	4.702	1.316	Reject
15	20	3.25	4.18	11	-1.83	2.9	0.05	$\mu_A \neq \mu_B$	3.573	2.045	Reject
16	12	2.29	1.06	18	0.66	1.08	0.10	$\mu_A < \mu_B$	4.079	-1.313	
17	19	3.78	1.04	19	1.88	1.09	0.01	$\mu_A \neq \mu_B$	5.497	2.719	Reject
18	17	-3.04	3.14	5	1.57	2.14	0.05	$\mu_A \neq \mu_B$	-3.054	2.086	Reject
19	8	-3.23	2.13	5	-5.49	2.1	0.01	$\mu_A < \mu_B$	1.871	-2.718	
20	5	-0.12	1.62	14	3.19	1.4	0.05	$\mu_A < \mu_B$	-4.367	-1.740	Reject
21	15	0.88	1.26	17	1.33	1.3	0.05	$\mu_A \neq \mu_B$	-0.991	2.042	
22	18	2.44	1.82	10	-4.66	1.96	0.05	$\mu_A < \mu_B$	9.628	-1.706	
23	9	1.68	1.33	19	3.23	1.33	0.05	$\mu_A > \mu_B$	-2.880	1.706	
24	7	0.72	0.90	9	0.37	1.41	0.10	$\mu_A < \mu_B$	0.570	-1.345	
25	6	5.30	1.97	5	-3.09	1.3	0.10	$\mu_A < \mu_B$	8.126	-1.383	

	n_A	$\bar{x}_A$	s_A	n_B	$\bar{x}_B$	s_B	α	H_a	T_{obs}	Crit.	Result
26	6	-0.18	0.88	12	-2.70	1.41	0.05	$\mu_A > \mu_B$	3.974	1.746	Reject
27	12	5.51	2.60	13	-1.19	2.24	0.10	$\mu_A > \mu_B$	6.919	1.319	Reject
28	14	1.39	1.17	11	4.22	1.7	0.10	$\mu_A > \mu_B$	-4.929	1.319	
29	18	-3.71	1.71	11	-3.61	2.11	0.01	$\mu_A \neq \mu_B$	-0.140	2.771	
30	14	-4.62	1.44	20	0.93	1.49	0.10	$\mu_A > \mu_B$	-10.835	1.309	
31	10	1.44	2.50	15	-1.71	1.45	0.05	$\mu_A > \mu_B$	3.998	1.714	Reject
32	10	-0.42	1.19	17	1.35	0.8	0.10	$\mu_A \neq \mu_B$	-4.632	1.708	Reject
33	12	2.76	1.31	12	-0.60	1.94	0.10	$\mu_A < \mu_B$	4.972	-1.321	
34	10	0.33	1.80	14	-0.18	2.4	0.10	$\mu_A < \mu_B$	0.566	-1.321	
35	13	4.93	2.76	19	-1.74	2.98	0.10	$\mu_A < \mu_B$	6.403	-1.310	
36	12	-0.12	1.81	20	5.50	1.25	0.01	$\mu_A > \mu_B$	-10.398	2.457	
37	20	-4.37	1.68	16	1.56	1.49	0.10	$\mu_A > \mu_B$	-11.057	1.307	
38	11	2.71	0.59	10	2.95	0.74	0.10	$\mu_A > \mu_B$	-0.826	1.328	
39	10	-0.59	0.84	20	1.71	0.67	0.01	$\mu_A > \mu_B$	-8.146	2.467	
40	14	-1.40	2.82	16	-3.96	2.52	0.10	$\mu_A > \mu_B$	2.626	1.313	Reject
41	18	4.26	0.78	5	-0.86	0.8	0.05	$\mu_A \neq \mu_B$	12.921	2.080	Reject
42	10	-1.78	1.66	20	1.52	2.27	0.10	$\mu_A \neq \mu_B$	-4.070	1.701	Reject
43	15	0.10	1.93	6	2.24	2.18	0.10	$\mu_A < \mu_B$	-2.216	-1.328	Reject
44	20	-3.39	2.08	16	3.93	2.49	0.05	$\mu_A \neq \mu_B$	-9.614	2.032	Reject
45	12	-4.45	1.29	13	-2.65	2.16	0.10	$\mu_A > \mu_B$	-2.502	1.319	
46	6	-1.93	1.98	8	0.92	1.71	0.10	$\mu_A < \mu_B$	-2.888	-1.356	Reject
47	17	-0.01	1.73	15	0.27	2.39	0.10	$\mu_A \neq \mu_B$	-0.383	1.697	
48	8	3.38	2.21	20	3.73	1.64	0.05	$\mu_A < \mu_B$	-0.462	-1.706	
49	6	-1.96	3.29	6	1.89	2.22	0.05	$\mu_A < \mu_B$	-2.376	-1.812	Reject
50	8	2.86	2.28	9	4.67	2.81	0.01	$\mu_A \neq \mu_B$	-1.446	2.947	