

MATH 204: PRINCIPLES OF STATISTICS 2
WINTER 2026
ANOVA TABLES FOR THE FACTORIAL DESIGN

1. **No interaction case**

Use the given information to complete the ANOVA tables in the format

SOURCE	DF	SS	MS	F
A	$a - 1$	SSA	MSA	F_A
B	$b - 1$	SSB	MSB	F_B
ERROR	$n - a - b + 1$	SSE	MSE	
TOTAL	$n - 1$	SS		

and complete the F-tests for the following analyses using statistical tables. As usual

- a is the number of levels of the Factor A
- b is the number of levels of the Factor B
- r is the number of replicates, which is the same for all factor level combinations.

Test at significance level $\alpha = 0.05$.

(i) Summary data: $a = 3, b = 3, r = 6$

$$SSA = 2.580 \quad SSB = 22.439 \quad SSE = 59.277$$

(ii) Summary data: $a = 4, b = 4, r = 1$

$$SSA = 1.937 \quad SSB = 11.905 \quad SSE = 50.369$$

(iii) Summary data: $a = 3, b = 3, r = 4$

$$SSA = 2.456 \quad SSB = 6.585 \quad SSE = 38.390$$

(iv) Summary data: $a = 8, b = 5, r = 5$

$$SSA = 0.157 \quad SSB = 17.823 \quad SSE = 59.614$$

(v) Summary data: $a = 6, b = 4, r = 5$

$$SSA = 0.860 \quad SSB = 10.979 \quad SSE = 45.247$$

(vi) Summary data: $a = 6, b = 5, r = 1$

$$SSA = 0.753 \quad SSB = 9.276 \quad SSE = 47.245$$

2. Interaction case

Use the given information to complete the ANOVA tables in the format

SOURCE	DF	SS	MS	F
A	$a - 1$	SSA	MSA	F_A
B	$b - 1$	SSB	MSB	F_B
INTERACTION	$(a - 1)(b - 1)$	SSI	MSI	F_{AB}
ERROR	$n - ab$	SSE	MSE	
TOTAL	$n - 1$	SS		

and complete the F-tests for the following analyses using statistical tables or software.

(i) Summary data: $a = 6, b = 4, r = 6$

$$SSA = 4.775 \quad SSB = 15.670 \quad SSI = 18.110 \quad SSE = 93.692$$

(ii) Summary data: $a = 4, b = 4, r = 6$

$$SSA = 8.475 \quad SSB = 22.252 \quad SSI = 6.890 \quad SSE = 110.774$$

(iii) Summary data: $a = 7, b = 3, r = 4$

$$SSA = 1.969 \quad SSB = 14.166 \quad SSI = 28.830 \quad SSE = 106.256$$

(iv) Summary data: $a = 5, b = 4, r = 4$

$$SSA = 16.563 \quad SSB = 21.385 \quad SSI = 15.379 \quad SSE = 112.241$$

(v) Summary data: $a = 5, b = 3, r = 2$

$$SSA = 2.145 \quad SSB = 15.784 \quad SSI = 31.162 \quad SSE = 119.847$$

(vi) Summary data: $a = 6, b = 5, r = 5$

$$SSA = 4.923 \quad SSB = 15.773 \quad SSI = 22.234 \quad SSE = 119.136$$

Solutions:

1. No interaction case

(i) ANOVA table

SOURCE	DF	SS	MS	F
A	2	2.580	1.290	1.066
B	2	22.439	11.220	9.275
ERROR	49	59.277	1.210	
TOTAL	53	84.296		

Factor A Critical value: 3.19 $\therefore$ Do not reject H_0

Factor B Critical value: 3.19 $\therefore$ Reject H_0

(ii) ANOVA table

SOURCE	DF	SS	MS	F
A	3	1.937	0.646	0.115
B	3	11.905	3.968	0.709
ERROR	9	50.369	5.597	
TOTAL	15	64.211		

Factor A Critical value: 3.86 $\therefore$ Do not reject H_0

Factor B Critical value: 3.86 $\therefore$ Do not reject H_0

(iii) ANOVA table

SOURCE	DF	SS	MS	F
A	2	2.456	1.228	0.992
B	2	6.585	3.293	2.659
ERROR	31	38.390	1.238	
TOTAL	35	47.431		

Factor A Critical value: 3.30 $\therefore$ Do not reject H_0

Factor B Critical value: 3.30 $\therefore$ Do not reject H_0

(iv) ANOVA table

SOURCE	DF	SS	MS	F
A	7	0.157	0.022	0.071
B	4	17.823	4.456	14.052
ERROR	188	59.614	0.317	
TOTAL	199	77.594		

Factor A Critical value: 2.06 $\therefore$ Do not reject H_0

Factor B Critical value: 2.42 $\therefore$ Reject H_0

(v) ANOVA table

SOURCE	DF	SS	MS	F
A	5	0.860	0.172	0.422
B	3	10.979	3.660	8.977
ERROR	111	45.247	0.408	
TOTAL	119	57.085		

Factor A Critical value: 2.30 $\therefore$ Do not reject H_0

Factor B Critical value: 2.69 $\therefore$ Reject H_0

(vi) ANOVA table

SOURCE	DF	SS	MS	F
A	5	0.753	0.151	0.064
B	4	9.276	2.319	0.982
ERROR	20	47.245	2.362	
TOTAL	29	57.275		

Factor A Critical value: 2.71 $\therefore$ Do not reject H_0

Factor B Critical value: 2.87 $\therefore$ Do not reject H_0

2. Interaction case

(i) ANOVA table

SOURCE	DF	SS	MS	F
A	5	4.775	0.955	1.223
B	3	15.670	5.223	6.690
INTERACTION	15	18.110	1.207	1.546
ERROR	120	93.692	0.781	
TOTAL	143	132.246		

Factor A Critical value: 2.29 $\therefore$ Do not reject H_0

Factor B Critical value: 2.68 $\therefore$ Reject H_0

Interaction Critical value: 1.75 $\therefore$ Do not reject H_0

(ii) ANOVA table

SOURCE	DF	SS	MS	F
A	3	8.475	2.825	2.040
B	3	22.252	7.417	5.357
INTERACTION	9	6.890	0.766	0.553
ERROR	80	110.774	1.385	
TOTAL	95	148.391		

Factor A Critical value: 2.72 $\therefore$ Do not reject H_0

Factor B Critical value: 2.72 $\therefore$ Reject H_0

Interaction Critical value: 2.00 $\therefore$ Do not reject H_0

(iii) ANOVA table

SOURCE	DF	SS	MS	F
A	6	1.969	0.328	0.195
B	2	14.166	7.083	4.200
INTERACTION	12	28.830	2.403	1.424
ERROR	63	106.256	1.687	
TOTAL	83	151.220		

Factor A Critical value: 2.25 $\therefore$ Do not reject H_0

Factor B Critical value: 3.14 $\therefore$ Reject H_0

Interaction Critical value: 1.91 $\therefore$ Do not reject H_0

(iv) ANOVA table

SOURCE	DF	SS	MS	F
A	4	16.563	4.141	2.213
B	3	21.385	7.128	3.810
INTERACTION	12	15.379	1.282	0.685
ERROR	60	112.241	1.871	
TOTAL	79	165.567		

Factor A Critical value: 2.53 $\therefore$ Do not reject H_0

Factor B Critical value: 2.76 $\therefore$ Reject H_0

Interaction Critical value: 1.92 $\therefore$ Do not reject H_0

(v) ANOVA table

SOURCE	DF	SS	MS	F
A	4	2.145	0.536	0.067
B	2	15.784	7.892	0.988
INTERACTION	8	31.162	3.895	0.488
ERROR	15	119.847	7.990	
TOTAL	29	168.939		

Factor A Critical value: 3.06 $\therefore$ Do not reject H_0

Factor B Critical value: 3.68 $\therefore$ Do not reject H_0

Interaction Critical value: 2.64 $\therefore$ Do not reject H_0

(vi) ANOVA table

SOURCE	DF	SS	MS	F
A	5	4.923	0.985	0.992
B	4	15.773	3.943	3.972
INTERACTION	20	22.234	1.112	1.120
ERROR	120	119.136	0.993	
TOTAL	149	162.066		

Factor A Critical value: 2.29 $\therefore$ Do not reject H_0

Factor B Critical value: 2.45 $\therefore$ Reject H_0

Interaction Critical value: 1.66 $\therefore$ Do not reject H_0