

MATH 204: PRINCIPLES OF STATISTICS 2
WINTER 2026

USING THE ANOVA F-TEST FOR THE COMPLETELY RANDOMIZED DESIGN

In a Completely Randomized Design (CRD) with k treatments, to test the hypothesis

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$$

H_a : At least two of the k treatment means are different

the key statistical quantities are

- MST - Mean Square for Treatments

$$MST = \frac{SST}{k - 1} = \frac{1}{k - 1} \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2$$

- MSE - Mean Square for Error

$$MSE = \frac{SSE}{n - k} = \frac{1}{n - k} \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2$$

The test statistic is

$$F = \frac{MST}{MSE}.$$

If H_0 is **true**, then

$$F \sim F_{k-1, n-k}$$

that is the F-distribution with $k - 1$ and $n - k$ degrees of freedom. The rejection region for a test at significance level α is the region **to the right** of the upper α point of this F -distribution (that is, using a **one-tailed** test).

We test under the assumptions of

- Normality for all samples
- Independence of the samples
- Common population variances in the k groups.

Suitable tables of the F-distribution can be found here:

Math204-Winter2026-FTable.pdf

Carry out the tests at the given α level.

Problem	k	n	SST	SSE	α
1	7	33	13.85	26.42	0.025
2	4	24	10.90	32.25	0.025
3	6	36	10.54	33.59	0.01
4	3	21	16.19	25.18	0.025
5	6	35	10.01	27.53	0.01
6	3	20	16.16	18.91	0.01
7	3	123	32.84	26.00	0.01
8	5	31	26.88	32.36	0.01
9	5	45	34.10	22.41	0.05
10	5	35	9.25	21.57	0.025
11	6	46	15.34	28.96	0.025
12	4	17	14.83	26.75	0.025
13	7	24	17.36	30.77	0.025
14	6	36	17.63	26.46	0.01
15	6	36	11.07	21.02	0.05
16	8	48	5.14	18.83	0.025
17	6	20	5.28	29.57	0.025
18	3	25	57.64	22.61	0.01
19	8	36	15.84	22.96	0.05
20	6	20	9.83	26.82	0.05
21	7	36	25.48	18.90	0.01
22	4	23	19.50	22.54	0.025
23	6	26	9.93	33.54	0.01
24	8	34	9.44	25.09	0.01
25	4	26	25.29	25.51	0.05
26	4	28	14.89	34.00	0.025
27	7	127	20.37	24.32	0.05
28	3	23	16.72	20.82	0.025
29	5	27	9.74	17.68	0.05
30	7	26	19.33	19.21	0.01

Solutions: We reject H_0 if $F_{obs} \geq C_R$, where $C_R = F_{\alpha, k-1, n-k}$ the upper α point of the F-distribution with $k - 1$ and $n - k$ degrees of freedom.

Problem	k	n	SST	SSE	α	MST	MSE	F_{obs}	C_R	Result
1	7	33	13.85	26.42	0.025	2.31	1.02	2.27	2.94	
2	4	24	10.90	32.25	0.025	3.63	1.61	2.25	3.86	
3	6	36	10.54	33.59	0.01	2.11	1.12	1.88	3.70	
4	3	21	16.19	25.18	0.025	8.10	1.40	5.79	4.56	Reject
5	6	35	10.01	27.53	0.01	2.00	0.95	2.11	3.73	
6	3	20	16.16	18.91	0.01	8.08	1.11	7.26	6.11	Reject
7	3	123	32.84	26.00	0.01	16.42	0.22	75.78	4.79	Reject
8	5	31	26.88	32.36	0.01	6.72	1.24	5.40	4.14	Reject
9	5	45	34.10	22.41	0.05	8.53	0.56	15.22	2.61	Reject
10	5	35	9.25	21.57	0.025	2.31	0.72	3.22	3.25	
11	6	46	15.34	28.96	0.025	3.07	0.72	4.24	2.90	Reject
12	4	17	14.83	26.75	0.025	4.94	2.06	2.40	4.35	
13	7	24	17.36	30.77	0.025	2.89	1.81	1.60	3.28	
14	6	36	17.63	26.46	0.01	3.53	0.88	4.00	3.70	Reject
15	6	36	11.07	21.02	0.05	2.21	0.70	3.16	2.53	Reject
16	8	48	5.14	18.83	0.025	0.73	0.47	1.56	2.62	
17	6	20	5.28	29.57	0.025	1.06	2.11	0.50	3.66	
18	3	25	57.64	22.61	0.01	28.82	1.03	28.04	5.72	Reject
19	8	36	15.84	22.96	0.05	2.26	0.82	2.76	2.36	Reject
20	6	20	9.83	26.82	0.05	1.97	1.92	1.03	2.96	
21	7	36	25.48	18.90	0.01	4.25	0.65	6.52	3.50	Reject
22	4	23	19.50	22.54	0.025	6.50	1.19	5.48	3.90	Reject
23	6	26	9.93	33.54	0.01	1.99	1.68	1.18	4.10	
24	8	34	9.44	25.09	0.01	1.35	0.96	1.40	3.42	
25	4	26	25.29	25.51	0.05	8.43	1.16	7.27	3.05	Reject
26	4	28	14.89	34.00	0.025	4.96	1.42	3.50	3.72	
27	7	127	20.37	24.32	0.05	3.40	0.20	16.76	2.18	Reject
28	3	23	16.72	20.82	0.025	8.36	1.04	8.03	4.46	Reject
29	5	27	9.74	17.68	0.05	2.44	0.80	3.03	2.82	Reject
30	7	26	19.33	19.21	0.01	3.22	1.01	3.19	3.94	