Estimation in ANOVA

Fitted values and predictions

Battery data: Interaction model

```
>Batt<-Batteries
>Batt$mat<-factor(as.character(Batteries$material),
                  labels=c("Ace","Pb","NiCa"))
```

```
>summary(aov(life ~ temp+mat+temp:mat,data=Batt))
             Df Sum Sq Mean Sq F value   Pr(>F)
temp          2  39083   19542  29.344 1.69e-07 ***
mat           2  10633    5317   7.983  0.00189 **
temp:mat      4   9438    2359   3.543  0.01897 *
Residuals    27  17981     666
---
```

Battery data: Interaction model

```
> summary(lm(life ~ temp+mat+temp:mat,data=Batt))
Coefficients:
                   Est. Std. Err t value Pr(>|t|)
(Intercept)      155.75   12.90  12.071 2.17e-12 ***
temp70           -38.50   18.25  -2.110   0.0443 *
temp125         -106.25   18.25  -5.823 3.38e-06 ***
matPb            -21.00   18.25  -1.151   0.2599
matNiCa          -11.75   18.25  -0.644   0.5251
temp70:matPb     -39.00   25.81  -1.511   0.1423
temp125:matPb     29.00   25.81   1.124   0.2710
temp70:matNiCa    40.25   25.81   1.560   0.1305
temp125:matNiCa   47.75   25.81   1.850   0.0752 .
---

Residual st. err.: 25.81 on 27 degrees of freedom
Multiple R-sq.: 0.7669, Adjusted R-sq.: 0.6978
F-stat.:  11.1 on 8 and 27 DF,  p-value: 8.606e-07
```

Battery data: Interaction model

```
> table(Batt$mat,Batt$temp)

       15 70 125
  Ace   4  4   4
  Pb    4  4   4
  NiCa  4  4   4

> xtabs(life ~ mat+temp,data=Batt)/
+                 table(Batt$mat,Batt$temp)

      temp
mat        15     70    125
  Ace  155.75 117.25  49.50
  Pb   134.75  57.25  57.50
  NiCa 144.00 145.75  85.50
```

Battery data: Predictions for interaction model

```
> newmat<-factor(rep(c(1,2,3),each=3),
+                          labels=c("Ace","Pb","NiCa"))
> newtemp<-factor(rep(c(1,2,3),3),
+                          labels=c("15","70","125"))
> newd<-data.frame(mat=newmat,temp=newtemp)

> newd
   mat temp
1  Ace   15
2  Ace   70
3  Ace  125
4   Pb   15
5   Pb   70
6   Pb  125
7 NiCa   15
8 NiCa   70
9 NiCa  125
```

Battery data: Predictions for interaction model

```
> #Predictions/fitted values
> newd$life<-predict(lm(life~temp+mat+temp:mat,
+                       data=Batt),newdata=newd)

> newd
   mat temp   life
1  Ace   15 155.75
2  Ace   70 117.25
3  Ace  125  49.50
4   Pb   15 134.75
5   Pb   70  57.25
6   Pb  125  57.50
7 NiCa   15 144.00
8 NiCa   70 145.75
9 NiCa  125  85.50
```

Battery data: Predictions for interaction model

```
> xtabs(life ~ mat+temp,data=newd)
      temp
mat         15     70    125
  Ace   155.75 117.25  49.50
  Pb    134.75  57.25  57.50
  NiCa 144.00 145.75  85.50

> #Observed means
> xtabs(life ~ mat+temp,data=Batt)/
+           table(Batt$mat,Batt$temp)
      temp
mat         15     70    125
  Ace   155.75 117.25  49.50
  Pb    134.75  57.25  57.50
  NiCa 144.00 145.75  85.50
```

Battery data: Main Effects Only Model

```
> summary(lm(life ~ temp+mat,data=Batt))

Coefficients:
                Est.  Std.Err. t value Pr(>|t|)
(Intercept)   147.08     11.08  13.271 2.52e-14 ***
temp70        -38.08     12.14  -3.137  0.00373 **
temp125       -80.67     12.14  -6.644 1.99e-07 ***
matPb         -24.33     12.14  -2.004  0.05385 .
matNiCa        17.58     12.14   1.448  0.15759
---

Residual std. err.: 29.74 on 31 degrees of freedom
Multiple R-sq.: 0.6445, Adjusted R-sq.: 0.5987
F-stat.: 14.05 on 4 and 31 DF,  p-value: 1.198e-06
```

Battery data: Predictions for Main Effects model

```
>newd$lifeADD<-predict(lm(life ~ temp+mat,
                         data=Batt),newdata=newd)
> newd
   mat temp   life   lifeADD
1  Ace   15 155.75 147.08333
2  Ace   70 117.25 109.00000
3  Ace  125  49.50  66.41667
4   Pb   15 134.75 122.75000
5   Pb   70  57.25  84.66667
6   Pb  125  57.50  42.08333
7 NiCa   15 144.00 164.66667
8 NiCa   70 145.75 126.58333
9 NiCa  125  85.50  84.00000
```

Battery data: Predictions for Main Effects model

```
> #Predictions/fitted values
> round(xtabs(lifeADD ~ mat+temp,data=newd),2)
      temp
mat       15     70    125
  Ace  147.08 109.00  66.42
  Pb   122.75  84.67  42.08
  NiCa 164.67 126.58  84.00

> #Observed means
> xtabs(life ~ mat+temp,data=Batt)/
+             table(Batt$mat,Batt$temp)
      temp
mat       15     70    125
  Ace  155.75 117.25  49.50
  Pb   134.75  57.25  57.50
  NiCa 144.00 145.75  85.50
```

Computing the predictions/fitted values

Consider a prediction for the combination

`temp=70` `mat=NiCa`

We combine terms from the coefficients tables to get this prediction. We add in one contribution from each term in the model. For the model

`temp+mat+temp:mat`

we combine **four** terms

`(Intercept) + temp70 + matNica + temp70:matNica`

corresponding to

$$\beta_1 + \beta_2^{\texttt{temp}} + \beta_3^{\texttt{mat}} + \gamma_{23}^{\texttt{temp:mat}}$$

that is, from slide 3,

$$155.75 + (-38.50) + (-11.75) + 40.25 = 145.75$$

Computing the predictions/fitted values

For the model

`temp+mat`

we combine **three** terms

`(Intercept) + temp70 + matNica`

corresponding to

$$\beta_1 + \beta_2^{\texttt{temp}} + \beta_3^{\texttt{mat}}$$

that is, from slide 8,

$$147.08 + (-38.08) + 17.58 = 126.58$$

Computing the predictions/fitted values

If we want a prediction at a **baseline** level of one or other factor, we add a contribution of **zero** for any term involving that level: for example, for

`temp=125` `mat=Ace`

- `temp+mat+temp:mat` - from slide 3

`(Intercept) + temp125 + matAce + temp125:matAce`

$$= 155.75 + (-106.25) + 0 + 0 = 49.50$$

- `temp+mat` - from slide 8

`(Intercept) + temp125 + matAce`

$$= 147.08 + (-80.67) + 0 = 66.41$$