2×2 tables and Odds Ratios

Data

Classification by two binary factors

		Exposure		
		Exposed	Not Exposed	Total
Outcome	Disease	n_{11}	n_{12}	$n_{1.}$
	No Disease	n_{21}	n_{22}	$n_{2.}$
	Total	$n_{.1}$	$n_{.2}$	n

The odds ratio **statistic** is

$$\widehat{\psi} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

Probabilities

Population Probabilities: Multinomial Experiment

		Exposure		
		Exposed	Not Exposed	Sum
Outcome	Disease	p_{11}	p_{12}	
	No Disease	p_{21}	p_{22}	
	Sum			1

Here

$$
\begin{aligned}
p_{11} &= \text{P[Disease and Exposed]} \\
p_{12} &= \text{P[Disease and Not Exposed]} \\
p_{21} &= \text{P[No Disease and Exposed]} \\
p_{22} &= \text{P[No Disease and Not Exposed]}
\end{aligned}
$$

In a designed experiment: conditioning on exposure

$$\begin{aligned}
\text{P[Disease | Exposed]} &= \frac{p_{11}}{p_{11}+p_{21}} \\
\text{P[No Disease | Exposed]} &= \frac{p_{21}}{p_{11}+p_{21}} \\
\text{P[Disease | Not Exposed]} &= \frac{p_{12}}{p_{12}+p_{22}} \\
\text{P[No Disease |Not Exposed]} &= \frac{p_{22}}{p_{12}+p_{22}}
\end{aligned}$$

Therefore

$$\begin{aligned}
\psi &= \frac{\text{P[Disease | Exposed]/P[No Disease | Exposed]}}{\text{P[Disease | Not Exposed]/P[No Disease |Not Exposed]}} \\
&= \frac{p_{11}p_{22}}{p_{12}p_{21}}
\end{aligned}$$

In a case-control study: conditioning on disease

$$\begin{aligned}
\text{P[Exposed | Disease]} &= \frac{p_{11}}{p_{11}+p_{12}} \\
\text{P[Not Exposed | Disease]} &= \frac{p_{12}}{p_{11}+p_{12}} \\
\text{P[Exposed |No Disease]} &= \frac{p_{21}}{p_{21}+p_{22}} \\
\text{P[Not Exposed |No Disease]} &= \frac{p_{22}}{p_{21}+p_{22}}
\end{aligned}$$

Therefore

$$\begin{aligned}
\psi &= \frac{\text{P[Exposed | Disease]/P[Exposed |No Disease]}}{\text{P[Not Exposed | Disease]/P[Not Exposed |No Disease]}} \\
&= \frac{p_{11}p_{22}}{p_{12}p_{21}}
\end{aligned}$$

Estimation

We may estimate

$$\begin{aligned}
\widehat{p}_{11} &= \frac{n_{11}}{n} \\
\widehat{p}_{12} &= \frac{n_{12}}{n} \\
\widehat{p}_{21} &= \frac{n_{21}}{n} \\
\widehat{p}_{22} &= \frac{n_{22}}{n}
\end{aligned}$$

so therefore

$$\widehat{\psi} = \frac{\widehat{p}_{11}\widehat{p}_{22}}{\widehat{p}_{12}\widehat{p}_{21}} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

Estimation

We may also estimate

$$\widehat{\mathrm{P}}[\text{ Disease} \mid \text{Exposed }] = \frac{n_{11}}{n_{11}+n_{21}}$$

$$\widehat{\mathrm{P}}[\text{No Disease} \mid \text{Exposed }] = \frac{n_{21}}{n_{11}+n_{21}}$$

$$\widehat{\mathrm{P}}[\text{ Disease} \mid \text{Not Exposed }] = \frac{n_{12}}{n_{12}+n_{22}}$$

$$\widehat{\mathrm{P}}[\text{No Disease} \mid \text{Not Exposed }] = \frac{n_{22}}{n_{12}+n_{22}}$$

and so on.