

MATH 203 Fall 2024: Final Formula Sheet

- **Numerical summaries:** Suppose $x_1, x_2, \dots, x_n$ be n data values.

► The *sample mean*, $\bar{x}$, is defined as:

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

► The *sample median*, m , is the number such that, if the data values were ordered in ascending order from smallest to largest then, m would be in the middle:

- * If n is odd, the sample median m is the number in the $(n+1)/2$ th position
- * If n is even, m is the average of the number in the $(n/2)$ th position and the number in the $(n/2 + 1)$ th position

► The *sample variance*, s^2 , is defined as:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2 \right) = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n(\bar{x})^2 \right)$$

- **Empirical rule:** For a perfectly symmetric, mound-shaped distribution, approximately

- 68% of the observations will lie in the range $(\bar{x} - s, \bar{x} + s)$,
- 95% of the observations will lie in the range $(\bar{x} - 2s, \bar{x} + 2s)$,
- 99.7% of the observations will lie in the range $(\bar{x} - 3s, \bar{x} + 3s)$.

- **Chebyshev's rule:** For any distribution and any number $k > 1$: at least

$$\left(1 - \frac{1}{k^2} \right) \times 100\%$$

of observations will fall into the interval $(\bar{x} - ks, \bar{x} + ks)$ regardless of the shape of the sample.

- **Quartiles and quantiles:**

- The first sample (*lower*) quartile is the number, q_L , such that 25% of the observations are $\leq q_L$
- The second sample quartile is the median, m , such that 50% of the observations are $\leq m$
- The third sample (*upper*) quartile is the number, q_U , such that 75% of the observations are $\leq q_U$
- For $0 \leq p \leq 1$, the p th *sample quantile* is the number, q_p , such that $(100 \times p)\%$ of the observations are $\leq q_p$.

$$q_L = q_{0.25} \quad m = q_{0.5} \quad q_U = q_{0.75}$$

► Interquartile range (IQR) is $q_U - q_L$.

- **Boxplot construction:**

- *Median line*
- *Box* (lower to upper quartile) or lower and upper hinge
- *Inner fences:* the upper inner fence is $q_U + 1.5 \times \text{IQR}$, the lower inner fence is $q_L - 1.5 \times \text{IQR}$.
- *Outer fences:* the upper outer fence is $q_U + 3 \times \text{IQR}$, the lower outer fence is $q_L - 3 \times \text{IQR}$.
- *Whiskers:* at largest/smallest data points less extreme than inner fences

- **Set theory:** Suppose S is a set, and A and B subsets of S .
 - ▶ the *intersection* of two sets A and B , $A \cap B$, is the collection of elements that are elements of *both* A and B .
 - ▶ the *union* of two sets A and B , $A \cup B$, is the collection of elements that are either in A , or in B , or in both A and B .
 - ▶ the *complement* of set A , A^c , is the collection of elements of S that are not elements of A .
 - ▶ We have

$$(A \cap B) \cap C = A \cap B \cap C$$

$$(A \cup B) \cup C = A \cup B \cup C$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- ▶ *Basic partition:* we have the partition of S via the four sets

$$A \cap B \quad A \cap B^c \quad A^c \cap B \quad A^c \cap B^c$$

- ▶ *de Morgan's Laws:*

$$A^c \cap B^c = (A \cup B)^c \quad A^c \cup B^c = (A \cap B)^c$$

- **Counting principles:**

- ▶ *Multiplication principle:* A sequence of k operations, in which operation i can result in n_i possible outcomes, can result in $n_1 \times n_2 \times \cdots \times n_k$ possible sequences of outcomes.
- ▶ *Permutations:* An ordered arrangement of r distinct objects is called a *permutation*. The number of ways of ordering r distinct objects taken from n without replacement is denoted by

$$P_r^n = \frac{n!}{(n-r)!}$$

- ▶ *Combinations:* The number of *combinations* of n objects taken r at a time (without replacement) is the number of subsets of size r that can be formed. This number is denoted

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

The number of binary sequences of length n containing r 1s is C_r^n .

- **Probability:** Suppose S is a sample space for an experiment, A and B are events.

- ▶ *The rules of probability:*

(I) $P(A) \geq 0$.

(II) $P(S) = 1$.

(III) If $A \cap B = \emptyset$, then

$$P(A \cup B) = P(A) + P(B).$$

► *Consequences of the rules:*

- (i) For any A , $P(A^c) = 1 - P(A)$.
- (ii) $P(\emptyset) = 0$.
- (iii) For any A , $P(A) \leq 1$.
- (iv) For two events A and B , if $A \subseteq B$, then

$$P(A) \leq P(B)$$

► *General Addition Rule:*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

► *Boole's Inequality:* For a collection of events $A_1, A_2, \dots, A_K$.

$$P\left(\bigcup_{k=1}^K A_k\right) \leq \sum_{k=1}^K P(A_k).$$

- **Conditional probability:** For two events A and B , the conditional probability of A given B is denoted $P(A|B)$, and is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

We consider this where $P(B) > 0$. It follows that $P(A \cap B) = P(A|B)P(B)$.

A and B are *independent* if $P(A|B) = P(A)$, equivalently if $P(A \cap B) = P(A)P(B)$.

- **Chain rule:** we have that

$$P(A_1 \cap A_2 \cap \dots \cap A_K) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \dots P(A_K|A_1 \cap A_2 \cap A_3 \cap \dots \cap A_{K-1}).$$

- **Probability Theorems:** For two events A and B

Theorem of Total Probability

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

Bayes Theorem:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

General versions: Suppose events $B_1, B_2, \dots, B_K$ form a partition of sample space S , and let $A \subseteq S$.

Provided all the conditional probabilities are well-defined, we have

► *Theorem of Total Probability:*

$$P(A) = \sum_{k=1}^K P(A|B_k)P(B_k).$$

► *Bayes Theorem:*

$$P(B_k|A) = \frac{P(A|B_k)P(B_k)}{P(A)} = \frac{P(A|B_k)P(B_k)}{\sum_{j=1}^K P(A|B_j)P(B_j)}.$$

- **Probability distributions:** For random variable Y taking values on a set $\mathcal{Y}$.

► *Discrete distributions:* We specify a discrete distribution using the

- * probability mass function (pmf) $p(y) = P(Y = y)$
- * the cumulative distribution function (cdf) $F(y) = P(Y \leq y)$

The pmf must satisfy two conditions: it must specify probabilities: that is

$$0 \leq p(y) \leq 1 \quad \text{for all } y$$

and must sum to 1 when evaluated at points in $\mathcal{Y}$.

► The *expectation* and *variance* of a discrete distribution are defined as

$$\mathbb{E}(Y) = \sum_{y \in \mathcal{Y}} y p(y) \quad \text{Var}(Y) = \mathbb{E}[(Y - \mu)^2] = \mathbb{E}(Y^2) - \{\mathbb{E}(Y)\}^2.$$

where $\mu = \mathbb{E}(Y)$. The *standard deviation* of Y is $\text{sd}(Y) = \sqrt{\text{Var}(Y)}$. If Y takes values on the finite set $\{y_1, y_2, \dots, y_n\}$ with probabilities $\{p(y_1), p(y_2), \dots, p(y_n)\}$, then

$$\mathbb{E}(Y) = \sum_{i=1}^n y_i p(y_i) \quad \mathbb{E}(Y^2) = \sum_{i=1}^n y_i^2 p(y_i)$$

and so on.

► *Continuous distributions:* We specify a continuous distribution using the probability density function (pdf), $f(y)$, or the cumulative distribution function (cdf) $F(y)$

$$F(y) = P(Y \leq y) = \int_{-\infty}^y f(t) dt.$$

The pdf must be non-negative, $f(y) \geq 0$, for all y and must integrate to 1.

- **Standard distributions:**

Distn.	$\mathcal{Y}$	Param.	pmf/pdf	cdf	$\mathbb{E}(Y)$	$\text{Var}(Y)$
Bernoulli	$\{0, 1\}$	p	$p^y(1-p)^{n-y}$		p	$p(1-p)$
<i>Number of successes in binary trial with success prob p</i>						
Binomial	$\{0, 1, 2, \dots, n\}$	p	$\binom{n}{y} p^y(1-p)^{n-y}$		np	$np(1-p)$
<i>Number of successes in n independent binary trials with success prob p</i>						
Poisson	$\{0, 1, 2, \dots\}$	λ	$\frac{e^{-\lambda} \lambda^y}{y!}$		λ	λ
<i>Limiting case of Binomial with n large, p small, and $\lambda = np$.</i>						
Geometric	$\{1, 2, 3, \dots\}$	p	$(1-p)^{y-1} p$	$1 - (1-p)^y$	$1/p$	$(1-p)/p^2$
<i>Number of trials until first success in independent binary trials with success prob p</i>						
Normal	$(-\infty, \infty)$	(μ, σ)	$\frac{1}{\sqrt{2\pi}\sigma^2} \exp\left\{-\frac{1}{2\sigma^2}(y-\mu)^2\right\}$		μ	σ^2

- **Properties of Normal random variables:**

- ▶ If $Y \sim \text{Normal}(\mu, \sigma^2)$, then

$$Z = \frac{Y - \mu}{\sigma} \sim \text{Normal}(0, 1)$$

so

$$P(Y \leq y) = P\left(Z \leq \frac{y - \mu}{\sigma}\right) = \Phi\left(\frac{y - \mu}{\sigma}\right).$$

where $\Phi(\cdot)$ is the standard Normal cdf.

- ▶ If $X_1 \sim \text{Normal}(\mu_1, \sigma_1^2)$ and $X_2 \sim \text{Normal}(\mu_2, \sigma_2^2)$ are independent, then

$$Y_1 = X_1 + X_2 \sim \text{Normal}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$$

and

$$Y_2 = X_1 - X_2 \sim \text{Normal}(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$$

- ▶ If $X_1, \dots, X_n$ are independent and identically distributed $\text{Normal}(\mu, \sigma^2)$ random variables, then

$$S_n = \sum_{i=1}^n X_i \sim \text{Normal}(n\mu, n\sigma^2)$$

and

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim \text{Normal}\left(\mu, \frac{\sigma^2}{n}\right)$$

- ▶ *Central Limit Theorem:* $X_1, \dots, X_n$ are independent and identically distributed random variables with expectation μ and variance σ^2 , then

$$S_n = \sum_{i=1}^n X_i \rightsquigarrow \text{Normal}(n\mu, n\sigma^2)$$

and

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \rightsquigarrow \text{Normal}\left(\mu, \frac{\sigma^2}{n}\right)$$

so we may approximate

$$P(S_n \leq s) \approx \Phi\left(\frac{s - n\mu}{\sqrt{n\sigma^2}}\right)$$

and

$$P(\bar{X} \leq x) \approx \Phi\left(\frac{x - \mu}{\sigma/\sqrt{n}}\right)$$

- **Estimation:**

- ▶ A point estimator $\hat{\theta}$ is unbiased for θ if $\mathbb{E}(\hat{\theta}) = \theta$.
- ▶ A point estimator of a population mean μ is the sample mean $\hat{\mu} = \bar{X}$.
- ▶ A point estimator of a population proportion p is the sample proportion $\hat{p} = X/n$.

- **Confidence Intervals:** Interval (L, R) is a $100(1 - \alpha)\%$ confidence interval (C. I.) for parameter θ if, were we to carry out the same study infinitely often under the same conditions, and each time compute a $100(1 - \alpha)\%$ C. I. using the same procedure, then $100(1 - \alpha)\%$ of these intervals would contain θ .

Reported Confidence Intervals always take the form

$$\left(\hat{\theta} - u_{\alpha/2} \times \text{s.e.}(\hat{\theta}), \hat{\theta} + u_{\alpha/2} \times \text{s.e.}(\hat{\theta}) \right)$$

- $\hat{\theta}$ is the point estimate
- $u_{\alpha/2}$ is the upper $\alpha/2$ point of the relevant (Normal or t-) distribution
- $\text{s.e.}(\hat{\theta})$ is the standard deviation of $\hat{\theta}$ in its sampling distribution (the *standard error*).

Parameter	Estimator	Estimate	s.e.	Notes
μ	$\bar{X}$	$\bar{x}$	$\sigma/\sqrt{n}$	σ known
			$s/\sqrt{n}$	σ unknown
p	X/n	$\hat{p} = x/n$	$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	large sample
$\mu_1 - \mu_2$	$\bar{X} - \bar{Y}$	$\bar{x} - \bar{y}$	$s_p \sqrt{\frac{1}{m} + \frac{1}{n}}$	small sample
			$\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$	large sample
$\mu_d = \mu_1 - \mu_2$	$\bar{D} = \bar{X} - \bar{Y}$	$\bar{d} = \bar{x} - \bar{y}$	$s_d/\sqrt{n}$	paired
$p_1 - p_2$	$X/m - Y/m$	$\hat{p}_1 - \hat{p}_2 = x/m - y/n$	$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{m} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}}$	large sample

Small sample cases with σ unknown must assume Normality of the data, and use the t-distribution. Large sample corresponds to $n \geq 30$, or $n\hat{p} \geq 15$, $n(1-\hat{p}) \geq 15$, with equivalent definitions in the two sample case. See the table on the next page for more details.

- **Hypothesis tests:** To test null hypothesis H_0 against alternative hypothesis H_a with Type 1 level α using test statistic T , use the Rejection Region and p-values are defined as follows:

$$\begin{array}{llll}
 H_a & H_a : \mu > \mu_0 & H_a : \mu < \mu_0 & H_a : \mu \neq \mu_0 \\
 \text{R. R.} & \{T \geq u_\alpha\} & \{T \leq -u_\alpha\} & \{|T| \geq u_{\alpha/2}\} \\
 \text{p-value} & P_0(T \geq T_{obs}) & P_0(T \leq T_{obs}) & 2P_0(T \geq |T_{obs}|)
 \end{array}$$

where u_α is the upper α point of the relevant (Normal or t-) distribution, T_{obs} is the observed value of the test statistic, and P_0 is the distribution of T assuming that H_0 is true. Testing for Binomial proportion p proceeds identically.

	Sample	Paired	Sample size	CI	Z-transform	H_0	T	Null distr.
μ	One	No	Small	μ	$\frac{\bar{X} - \mu}{S/\sqrt{n}}$	$\mu = \mu_0$	$\frac{\bar{X} - \mu_0}{S/\sqrt{n}}$	t_{n-1}
			Large					$N(0,1)$
	Two	No	Small	$\mu_1 - \mu_2$	$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{m} + \frac{1}{n}}}$	$\mu_1 = \mu_2$	$\frac{\bar{X} - \bar{Y}}{S_p \sqrt{\frac{1}{m} + \frac{1}{n}}}$	t_{m+n-2}
			Large	$\mu_1 - \mu_2$	$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}$	$\mu_1 = \mu_2$	$\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}$	$N(0,1)$
p		Yes	Small	$\mu_d = \mu_1 - \mu_2$	$\frac{\bar{D} - \mu_d}{S_d/\sqrt{n}}$	$\mu_1 = \mu_2$	$\frac{\bar{D}}{S_d/\sqrt{n}}$	t_{n-1}
			Large					$N(0,1)$
	One	No	Large	p	$\frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$	$p = p_0$	$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$N(0,1)$
	Two	No	Large	$p_1 - p_2$	$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{m} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}}}$	$p_1 = p_2$	$\frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\hat{p}(1-\hat{p})}{m} + \frac{1}{n}}}$	$N(0,1)$
$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$								
$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$				$\bar{X}, \bar{Y}$	One sample: $\hat{p} = \frac{1}{n} \sum_{i=1}^n X_i$			
$S_d^2 = \frac{1}{n-1} \sum_{i=1}^n (D_i - \bar{D})^2$				S_1^2, S_2^2	Two sample: $\hat{p}_1 = \frac{1}{m} \sum_{i=1}^m X_i, \hat{p}_2 = \frac{1}{n} \sum_{i=1}^n Y_i$			
				$S_p^2 = \frac{(m-1)S_1^2 + (n-1)S_2^2}{(m+n-2)}$	$\hat{p} = \frac{1}{n+m} \left(\sum_{i=1}^m X_i + \sum_{i=1}^n Y_i \right)$			

The Normal table displays the standard Normal cdf

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z f(t) dt$$

where t is the dummy/integrating variable, and $f(\cdot)$ is the standard Normal pdf.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Due to the symmetry of the standard Normal pdf, for $z < 0$

$$\Phi(z) = P(Z \leq z) = P(Z > -z) = 1 - P(Z \leq -z) = 1 - \Phi(-z).$$

Table 1: Upper a points of t_ν distribution

ν	a							
	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.001
4	1.5332	2.1318	2.7764	2.9985	3.7469	4.6041	5.5976	7.1732
5	1.4759	2.0150	2.5706	2.7565	3.3649	4.0321	4.7733	5.8934
6	1.4398	1.9432	2.4469	2.6122	3.1427	3.7074	4.3168	5.2076
7	1.4149	1.8946	2.3646	2.5168	2.9980	3.4995	4.0293	4.7853
8	1.3968	1.8595	2.3060	2.4490	2.8965	3.3554	3.8325	4.5008
9	1.3830	1.8331	2.2622	2.3984	2.8214	3.2498	3.6897	4.2968
10	1.3722	1.8125	2.2281	2.3593	2.7638	3.1693	3.5814	4.1437
11	1.3634	1.7959	2.2010	2.3281	2.7181	3.1058	3.4966	4.0247
12	1.3562	1.7823	2.1788	2.3027	2.6810	3.0545	3.4284	3.9296
13	1.3502	1.7709	2.1604	2.2816	2.6503	3.0123	3.3725	3.8520
14	1.3450	1.7613	2.1448	2.2638	2.6245	2.9768	3.3257	3.7874
15	1.3406	1.7531	2.1314	2.2485	2.6025	2.9467	3.2860	3.7328
16	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208	3.2520	3.6862
17	1.3334	1.7396	2.1098	2.2238	2.5669	2.8982	3.2224	3.6458
18	1.3304	1.7341	2.1009	2.2137	2.5524	2.8784	3.1966	3.6105
19	1.3277	1.7291	2.0930	2.2047	2.5395	2.8609	3.1737	3.5794
20	1.3253	1.7247	2.0860	2.1967	2.5280	2.8453	3.1534	3.5518
21	1.3232	1.7207	2.0796	2.1894	2.5176	2.8314	3.1352	3.5272
22	1.3212	1.7171	2.0739	2.1829	2.5083	2.8188	3.1188	3.5050
23	1.3195	1.7139	2.0687	2.1770	2.4999	2.8073	3.1040	3.4850
24	1.3178	1.7109	2.0639	2.1715	2.4922	2.7969	3.0905	3.4668
25	1.3163	1.7081	2.0595	2.1666	2.4851	2.7874	3.0782	3.4502
26	1.3150	1.7056	2.0555	2.1620	2.4786	2.7787	3.0669	3.4350
27	1.3137	1.7033	2.0518	2.1578	2.4727	2.7707	3.0565	3.4210
28	1.3125	1.7011	2.0484	2.1539	2.4671	2.7633	3.0469	3.4082
29	1.3114	1.6991	2.0452	2.1503	2.4620	2.7564	3.0380	3.3962
30	1.3104	1.6973	2.0423	2.1470	2.4573	2.7500	3.0298	3.3852
∞	1.2816	1.6449	1.9600	2.0537	2.3263	2.5758	2.8070	3.0902

This table displays upper a points for the t-distribution, that is, the values $t_{a,\nu}$ such that

$$P(T \leq t_{a,\nu}) = 1 - a \quad \text{or equivalently} \quad P(T > t_{a,\nu}) = a$$

where T is a random variable having a t-distribution with ν degrees of freedom; we fix a , and solve

$$\int_x^\infty f(t) dt = a$$

for x : the solution is $x = t_{a,\nu}$. The $\nu = \infty$ case corresponds to the standard Normal, and may be used when $\nu > 30$.