

MATH 203: PRINCIPLES OF STATISTICS 1
FALL 2024

EXERCISES ON HYPOTHESIS TESTING IN ONE SAMPLE PROBLEMS

Please try the questions before inspecting the solutions.

1. $n = 19, \bar{x} = 2.24, s = 2.83$. Test

$$H_0 : \mu = 2.4$$

$$H_a : \mu \neq 2.4$$

with $\alpha = 0.05$.

2. $n = 25, \bar{x} = 2.39, s = 1.03$. Test

$$H_0 : \mu = 1.9$$

$$H_a : \mu \neq 1.9$$

with $\alpha = 0.1$.

3. $n = 24, \bar{x} = 4.05, s = 1.28$. Test

$$H_0 : \mu = 3.4$$

$$H_a : \mu < 3.4$$

with $\alpha = 0.01$.

4. $n = 27, \bar{x} = 4.5, s = 2.52$. Test

$$H_0 : \mu = 4.1$$

$$H_a : \mu > 4.1$$

with $\alpha = 0.01$.

5. $n = 16, \bar{x} = -3.28, s = 2$. Test

$$H_0 : \mu = -3.7$$

$$H_a : \mu > -3.7$$

with $\alpha = 0.05$.

6. $n = 21, \bar{x} = 2.16, \sigma = 1.41$. Test

$$H_0 : \mu = 1.7$$

$$H_a : \mu > 1.7$$

with $\alpha = 0.1$.

7. $n = 12, \bar{x} = -4.93, \sigma = 1.54$. Test

$$H_0 : \mu = -5.2$$

$$H_a : \mu > -5.2$$

with $\alpha = 0.05$.

8. $n = 16, \bar{x} = 0.37, \sigma = 3.04$. Test

$$H_0 : \mu = 0.9$$

$$H_a : \mu \neq 0.9$$

with $\alpha = 0.1$.

9. $n = 29, \bar{x} = 3.14, \sigma = 1.72$. Test

$$H_0 : \mu = 3.2$$

$$H_a : \mu > 3.2$$

with $\alpha = 0.01$.

10. $n = 5, \bar{x} = -1.71, \sigma = 1.64$. Test

$$H_0 : \mu = -1.9$$

$$H_a : \mu > -1.9$$

with $\alpha = 0.05$.

11. $n = 47, \bar{x} = 1.62, s = 0.49$. Test

$$H_0 : \mu = -4.7$$

$$H_a : \mu > -4.7$$

with $\alpha = 0.05$.

12. $n = 74, \bar{x} = 0.53, s = 0.55$. Test

$$H_0 : \mu = 2.3$$

$$H_a : \mu > 2.3$$

with $\alpha = 0.05$.

13. $n = 44, \bar{x} = -19.44, s = 17.24$. Test

$$H_0 : \mu = -0.5$$

$$H_a : \mu < -0.5$$

with $\alpha = 0.05$.

14. $n = 74, \bar{x} = 0.49, s = 0.81$. Test

$$H_0 : \mu = 1.6$$

$$H_a : \mu > 1.6$$

with $\alpha = 0.01$.

15. $n = 66, \bar{x} = -0.36, s = 0.76$. Test

$$H_0 : \mu = 1.5$$

$$H_a : \mu < 1.5$$

with $\alpha = 0.1$.

16. $X \sim \text{Binomial}(n, p)$, with $n = 103$ and $x = 22$. Test

$$H_0 : p = 0.24$$

$$H_a : p > 0.24$$

with $\alpha = 0.05$.

17. $X \sim \text{Binomial}(n, p)$, with $n = 200$ and $x = 89$. Test

$$H_0 : p = 0.79$$

$$H_a : p \neq 0.79$$

with $\alpha = 0.01$.

18. $X \sim \text{Binomial}(n, p)$, with $n = 151$ and $x = 63$. Test

$$H_0 : p = 0.45$$

$$H_a : p > 0.45$$

with $\alpha = 0.05$.

19. $X \sim \text{Binomial}(n, p)$, with $n = 89$ and $x = 40$. Test

$$H_0 : p = 0.74$$

$$H_a : p \neq 0.74$$

with $\alpha = 0.01$.

20. $X \sim \text{Binomial}(n, p)$, with $n = 144$ and $x = 26$. Test

$$H_0 : p = 0.29$$

$$H_a : p \neq 0.29$$

with $\alpha = 0.01$.

Solutions:

1. The 0.05 Rejection Region is $\{|T| \geq t_{\alpha/2, n-1}\} = \{|T| \geq t_{0.025, 18}\} = \{|T| \geq 2.1009\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.24 - (2.4)}{2.83/\sqrt{19}} = -0.246$$

so we Do Not Reject H_0 in favour of H_a .

2. The 0.1 Rejection Region is $\{|T| \geq t_{\alpha/2, n-1}\} = \{|T| \geq t_{0.05, 24}\} = \{|T| \geq 1.7109\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.39 - (1.9)}{1.03/\sqrt{25}} = 2.379$$

so we Reject H_0 in favour of H_a .

3. The 0.01 Rejection Region is $\{T \leq -t_{\alpha, n-1}\} = \{T \leq -t_{0.01, 23}\} = \{T \leq -2.4999\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4.05 - (3.4)}{1.28/\sqrt{24}} = 2.488$$

so we Do Not Reject H_0 in favour of H_a .

4. The 0.01 Rejection Region is $\{T \geq t_{\alpha, n-1}\} = \{T \geq t_{0.01, 26}\} = \{T \geq 2.4786\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4.5 - (4.1)}{2.52/\sqrt{27}} = 0.825$$

so we Do Not Reject H_0 in favour of H_a .

5. The 0.05 Rejection Region is $\{T \geq t_{\alpha, n-1}\} = \{T \geq t_{0.05, 15}\} = \{T \geq 1.7531\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{-3.28 - (-3.7)}{2/\sqrt{16}} = 0.84$$

so we Do Not Reject H_0 in favour of H_a .

6. The 0.1 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.1}\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.16 - (1.7)}{1.41/\sqrt{21}} = 1.495$$

so we Reject H_0 in favour of H_a .

7. The 0.05 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.05}\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{-4.93 - (-5.2)}{1.54/\sqrt{12}} = 0.607$$

so we Do Not Reject H_0 in favour of H_a .

8. The 0.1 Rejection Region is $\{|T| \geq z_{\alpha/2}\} = \{|T| \geq z_{0.05}\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{0.37 - (0.9)}{3.04/\sqrt{16}} = -0.697$$

so we Do Not Reject H_0 in favour of H_a .

9. The 0.01 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.01}\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.14 - (3.2)}{1.72/\sqrt{29}} = -0.188$$

so we Do Not Reject H_0 in favour of H_a .

10. The 0.05 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.05}\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{-1.71 - (-1.9)}{1.64/\sqrt{5}} = 0.259$$

so we Do Not Reject H_0 in favour of H_a .

11. The 0.05 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.05}\} = \{T \geq 1.6449\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.62 - (-4.7)}{0.49/\sqrt{47}} = 88.424$$

so we Reject H_0 in favour of H_a .

12. The 0.05 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.05}\} = \{T \geq 1.6449\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.53 - (2.3)}{0.55/\sqrt{74}} = -27.684$$

so we Do Not Reject H_0 in favour of H_a .

13. The 0.05 Rejection Region is $\{T \leq -z_{\alpha}\} = \{T \leq -z_{0.05}\} = \{T \leq -1.6449\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{-19.44 - (-0.5)}{17.24/\sqrt{44}} = -7.287$$

so we Reject H_0 in favour of H_a .

14. The 0.01 Rejection Region is $\{T \geq z_{\alpha}\} = \{T \geq z_{0.01}\} = \{T \geq 2.3263\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.49 - (1.6)}{0.81/\sqrt{74}} = -11.788$$

so we Do Not Reject H_0 in favour of H_a .

15. The 0.1 Rejection Region is $\{T \leq -z_{\alpha}\} = \{T \leq -z_{0.1}\} = \{T \leq -1.2816\}$.

$$T_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{-0.36 - (1.5)}{0.76/\sqrt{66}} = -19.883$$

so we Reject H_0 in favour of H_a .

16. The 0.05 Rejection Region is $\{T \geq z_\alpha\} = \{T \geq z_{0.05}\} = \{T \geq 1.6449\}$.

$$T_{obs} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.214 - (0.24)}{\sqrt{0.24 \times 0.76/103}} = -0.628$$

so we Do Not Reject H_0 in favour of H_a .

17. The 0.01 Rejection Region is $\{|T| \geq z_{\alpha/2}\} = \{|T| \geq z_{0.005}\} = \{|T| \geq 2.5758\}$.

$$T_{obs} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.445 - (0.79)}{\sqrt{0.79 \times 0.21/200}} = -11.979$$

so we Reject H_0 in favour of H_a .

18. The 0.05 Rejection Region is $\{T \geq z_\alpha\} = \{T \geq z_{0.05}\} = \{T \geq 1.6449\}$.

$$T_{obs} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.417 - (0.45)}{\sqrt{0.45 \times 0.55/151}} = -0.81$$

so we Do Not Reject H_0 in favour of H_a .

19. The 0.01 Rejection Region is $\{|T| \geq z_{\alpha/2}\} = \{|T| \geq z_{0.005}\} = \{|T| \geq 2.5758\}$.

$$T_{obs} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.449 - (0.74)}{\sqrt{0.74 \times 0.26/89}} = -6.25$$

so we Reject H_0 in favour of H_a .

20. The 0.01 Rejection Region is $\{|T| \geq z_{\alpha/2}\} = \{|T| \geq z_{0.005}\} = \{|T| \geq 2.5758\}$.

$$T_{obs} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.181 - (0.29)}{\sqrt{0.29 \times 0.71/144}} = -2.894$$

so we Reject H_0 in favour of H_a .