

MATH 203: PRINCIPLES OF STATISTICS 1
FALL 2024

EXERCISES ON CONFIDENCE INTERVALS

Please try the questions before inspecting the solutions on p2 and p3.

1. Scenario 1: $n = 13$, $\bar{x} = -2.22$, $s = 2.25$. Compute the 95% C.I.
2. Scenario 1: $n = 8$, $\bar{x} = -3.36$, $s = 1.5$. Compute the 90% C.I.
3. Scenario 1: $n = 12$, $\bar{x} = -1.33$, $s = 2.61$. Compute the 99% C.I.
4. Scenario 1: $n = 5$, $\bar{x} = 1.09$, $s = 1.81$. Compute the 95% C.I.
5. Scenario 1: $n = 17$, $\bar{x} = 0.76$, $s = 3.29$. Compute the 80% C.I.
6. Scenario 2: $n = 18$, $\bar{x} = -1.46$, $\sigma = 2.62$. Compute the 90% C.I.
7. Scenario 2: $n = 16$, $\bar{x} = -3.12$, $\sigma = 2.47$. Compute the 95% C.I.
8. Scenario 2: $n = 27$, $\bar{x} = -2.71$, $\sigma = 1.83$. Compute the 90% C.I.
9. Scenario 2: $n = 9$, $\bar{x} = -3.47$, $\sigma = 2.71$. Compute the 90% C.I.
10. Scenario 2: $n = 26$, $\bar{x} = -2.82$, $\sigma = 2.93$. Compute the 95% C.I.
11. Scenario 3: $n = 59$, $\bar{x} = -14.48$, $s = 15.94$. Compute the 80% C.I.
12. Scenario 3: $n = 54$, $\bar{x} = 1.19$, $s = 0.55$. Compute the 95% C.I.
13. Scenario 3: $n = 57$, $\bar{x} = 0.92$, $s = 0.45$. Compute the 95% C.I.
14. Scenario 3: $n = 56$, $\bar{x} = -0.81$, $s = 0.95$. Compute the 90% C.I.
15. Scenario 3: $n = 60$, $\bar{x} = 0.44$, $s = 0.64$. Compute the 80% C.I.
16. Scenario 4: $X \sim \text{Binomial}(n, p)$, with $n = 129$ and $x = 22$. Compute the 95% C.I.
17. Scenario 4: $X \sim \text{Binomial}(n, p)$, with $n = 178$ and $x = 23$. Compute the 80% C.I.
18. Scenario 4: $X \sim \text{Binomial}(n, p)$, with $n = 139$ and $x = 69$. Compute the 80% C.I.
19. Scenario 4: $X \sim \text{Binomial}(n, p)$, with $n = 180$ and $x = 75$. Compute the 95% C.I.
20. Scenario 4: $X \sim \text{Binomial}(n, p)$, with $n = 193$ and $x = 38$. Compute the 95% C.I.

Solutions

1. The 95% C.I. is

$$\left(\bar{x} - t_{0.025,12} \frac{s}{\sqrt{n}}, \bar{x} + t_{0.025,12} \frac{s}{\sqrt{n}} \right) = -2.22 \pm 2.1788 \frac{2.25}{\sqrt{13}} = (-3.5797, -0.8603)$$

2. The 90% C.I. is

$$\left(\bar{x} - t_{0.05,7} \frac{s}{\sqrt{n}}, \bar{x} + t_{0.05,7} \frac{s}{\sqrt{n}} \right) = -3.36 \pm 1.8946 \frac{1.5}{\sqrt{8}} = (-4.3648, -2.3552)$$

3. The 99% C.I. is

$$\left(\bar{x} - t_{0.005,11} \frac{s}{\sqrt{n}}, \bar{x} + t_{0.005,11} \frac{s}{\sqrt{n}} \right) = -1.33 \pm 3.1058 \frac{2.61}{\sqrt{12}} = (-3.67, 1.01)$$

4. The 95% C.I. is

$$\left(\bar{x} - t_{0.025,4} \frac{s}{\sqrt{n}}, \bar{x} + t_{0.025,4} \frac{s}{\sqrt{n}} \right) = 1.09 \pm 2.7764 \frac{1.81}{\sqrt{5}} = (-1.1574, 3.3374)$$

5. The 80% C.I. is

$$\left(\bar{x} - t_{0.1,16} \frac{s}{\sqrt{n}}, \bar{x} + t_{0.1,16} \frac{s}{\sqrt{n}} \right) = 0.76 \pm 1.3368 \frac{3.29}{\sqrt{17}} = (-0.3067, 1.8267)$$

6. The 90% C.I. is

$$\left(\bar{x} - z_{0.05} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{0.05} \frac{\sigma}{\sqrt{n}} \right) = -1.46 \pm 1.6449 \frac{2.62}{\sqrt{18}} = (-2.4758, -0.4442)$$

7. The 95% C.I. is

$$\left(\bar{x} - z_{0.025} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{0.025} \frac{\sigma}{\sqrt{n}} \right) = -3.12 \pm 1.96 \frac{2.47}{\sqrt{16}} = (-4.3303, -1.9097)$$

8. The 90% C.I. is

$$\left(\bar{x} - z_{0.05} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{0.05} \frac{\sigma}{\sqrt{n}} \right) = -2.71 \pm 1.6449 \frac{1.83}{\sqrt{27}} = (-3.2893, -2.1307)$$

9. The 90% C.I. is

$$\left(\bar{x} - z_{0.05} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{0.05} \frac{\sigma}{\sqrt{n}} \right) = -3.47 \pm 1.6449 \frac{2.71}{\sqrt{9}} = (-4.9559, -1.9841)$$

10. The 95% C.I. is

$$\left(\bar{x} - z_{0.025} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{0.025} \frac{\sigma}{\sqrt{n}} \right) = -2.82 \pm 1.96 \frac{2.93}{\sqrt{26}} = (-3.9463, -1.6937)$$

Solutions

11. The 80% C.I. is

$$\left(\bar{x} - z_{0.1} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.1} \frac{s}{\sqrt{n}} \right) = -14.48 \pm 1.2816 \frac{15.94}{\sqrt{59}} = (-17.1396, -11.8204)$$

12. The 95% C.I. is

$$\left(\bar{x} - z_{0.025} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.025} \frac{s}{\sqrt{n}} \right) = 1.19 \pm 1.96 \frac{0.55}{\sqrt{54}} = (1.0433, 1.3367)$$

13. The 95% C.I. is

$$\left(\bar{x} - z_{0.025} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.025} \frac{s}{\sqrt{n}} \right) = 0.92 \pm 1.96 \frac{0.45}{\sqrt{57}} = (0.8032, 1.0368)$$

14. The 90% C.I. is

$$\left(\bar{x} - z_{0.05} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.05} \frac{s}{\sqrt{n}} \right) = -0.81 \pm 1.6449 \frac{0.95}{\sqrt{56}} = (-1.0188, -0.6012)$$

15. The 80% C.I. is

$$\left(\bar{x} - z_{0.1} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.1} \frac{s}{\sqrt{n}} \right) = 0.44 \pm 1.2816 \frac{0.64}{\sqrt{60}} = (0.3341, 0.5459)$$

16. The 95% C.I. is

$$\left(\hat{p} - z_{0.025} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + z_{0.025} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \right) = 0.1705 \pm 1.96 \frac{0.3761}{\sqrt{129}} = (0.1056, 0.2354)$$

17. The 80% C.I. is

$$\left(\hat{p} - z_{0.1} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + z_{0.1} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \right) = 0.1292 \pm 1.2816 \frac{0.3354}{\sqrt{178}} = (0.097, 0.1614)$$

18. The 80% C.I. is

$$\left(\hat{p} - z_{0.1} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + z_{0.1} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \right) = 0.4964 \pm 1.2816 \frac{0.5}{\sqrt{139}} = (0.4421, 0.5508)$$

19. The 95% C.I. is

$$\left(\hat{p} - z_{0.025} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + z_{0.025} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \right) = 0.4167 \pm 1.96 \frac{0.493}{\sqrt{180}} = (0.3446, 0.4887)$$

20. The 95% C.I. is

$$\left(\hat{p} - z_{0.025} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + z_{0.025} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \right) = 0.1969 \pm 1.96 \frac{0.3976}{\sqrt{193}} = (0.1408, 0.253)$$