189-245A: Honors Algebra I

Practice Midterm Exam

Reading Break

1. State (without proving it) the Euclidean division algorithm in $\mathbf{Z}$. Using only this division algorithm and the well-ordering principle, prove that any common divisor of two non-zero integers a and b necessarily *divides* the gcd of a and b.

2. Compute the greatest common divisor of the elements $x^7+x^6+x^4+x^3+1$ and $x^8+x^6+x^5+x^4+x^3+x^2+1$ in the ring $\mathbf{Z}_2[x]$.

3. Let p be an odd prime. Show that the ring $\mathbf{Z}/p\mathbf{Z}$ contains a square root of -1 if and only if p is of the form $1+4k$.

4. Show that the ring $\mathbf{Z}_3[x]/(x^3+x^2+2)$ is a field with 27 elements. (For this you may quote any theorem that was proved in class.)