# 189-245A: Honours Algebra 1

# Practice Midterm Exam

## 225 Study Break

*Each of the four questions below is worth 25 points. No calculators or outside materials are allowed during the exam.*

1. Let $i$ be the complex number satisfying $i^2 = -1$. Compute $(1+i)^{100}$.

2. Let $a$ and $b$ be non-zero integers, and let $L$ be the set of all *strictly positive* linear combinations of $a$ and $b$:

$$L = \{ra + sb, \text{ with } r, s \in \mathbf{Z} \text{ and } ra + sb > 0\}.$$

(a) Show that the smallest element of $L$ divides $a$ and $b$.
(b) Show that this smallest element is the gcd of $a$ and $b$.

3. Compute the reduced residue modulo $N$ (i.e., the unique integer $0 \le x \le N-1$ with $x \equiv a \pmod{N}$) of the integer

$$a = 7^{1319845934875198347586734589234239820923498346523453 1}$$

for the following values of $N$.
(a) $N = 11$;
(b) $N = 5$;
(c) $N = 55$.

4. Solve the following congruence equations (making sure you list all the distinct solutions in $\mathbf{Z}/N\mathbf{Z}$).
(a) $5x = 2 \pmod{11}$.
(b) $10x = 4 \pmod{22}$.
(c) $10x = 3 \pmod{22}$.