

189-245A: Honors Algebra 1

Midterm Exam

Friday, October 24

Each of the four questions below is worth 25 points. Calculators or outside materials are strictly forbidden during the exam.

1. Compute the remainder in the division of $96^{7834061028736012983765103874601602}$ by the prime number 101.

2. Let a and $b \in \mathbf{Z}$ be two integers whose only common divisors are the units 1 and -1 . For any $c \in \mathbf{Z}$, prove that a divides bc if and only if a divides c .

3. Let n be a positive integer which is ≤ 120 and is not divisible by 2, 3, 5 or 7. Show that n is a prime number.

4. (a) What is the cardinality of the ring $\mathbf{Z}/5\mathbf{Z}[x]/(x^3 + x + 1)$?

(b) Show that this ring is a field, i.e., that every non-zero element in it admits a multiplicative inverse. (You can, of course, use any theorem that we proved in class but carefully state what you are using.)