189-726B: Modular Forms II
Assignment 10

Due: Wednesday, April 26

1. If f and g are modular forms of weight k and ℓ respectively, on some congruence group $\Gamma_0(N)$, and having fourier coefficients in a field K, show that the expression

$$[f, g] := \frac{1}{2\pi i}(\ell f' g - k f g')$$

is a modular form of weight $k + \ell + 2$ with fourier coefficients in K. (Here, f' is just the usual derivative of f with respect to the variable τ.)

2. The operation $(f, g) \mapsto [f, g]$ is called the *Rankin-Cohen bracket*. Show that this operation endows the space $M := \oplus_{k \geq 0} M_k$ of classical modular forms (with coefficients in a field K) with the structure of a (graded) Lie algebra over K.

3. Compute the Rankin-Cohen bracket $[Q, R]$ where Q and R are the usual normalised Eisenstein series of weights 4 and 6 respectively.