189-726B: Modular Forms II
Assignment 4

Due: Wednesday, February 6

1. Show that the action of the Hecke operator T_n and of the complex conjugation c on the lattice $\Lambda = H_1(X_0(N)(\mathbf{C}), \mathbf{Z}) \subset S_2(\Gamma_0(N))^\vee$ that were defined in class commute with each other.

2. Prove the assertion made in class that the set of orbits for the action of $\Gamma_0(N)$ on the set of ordered pairs of adjacent cusps in $\mathbf{P}_1(\mathbf{Q})$ is in natural bijection with $\mathbf{P}_1(\mathbf{Z}/N\mathbf{Z})$.

3. Compute the space $\mathcal{M}^{\Gamma_0(11)}$ of modular symbols relative to the group $\Gamma_0(11)$. Show that there is exactly one cusp form f on this group, and compute $a_2(f)$ and $a_3(f)$ using the modular symbol algorithm. Check that your answer agrees with

$$f(q) = q \prod_n (1 - q^n)^2 (1 - q^{11n})^2.$$