From word to sentence: a pregroup analysis of the object pronoun *who(m)*.

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We explore a computational algebraic approach to grammar via *pregroups*, that is, partially ordered monoids in which each element has both a left and a right *adjoint*. Grammatical judgements are formed with the help of calculations on *types*. These are elements of the free pregroup generated by a partially ordered set of *basic types*, which are assigned to words, here of English. We concentrate on the object pronoun *who(m)*.

1. Historical introduction.

Categorial grammar is an attempt to describe the structure of a natural language by assigning certain types, formerly known as *categories*, to the words of the language. These types live in an algebraic or logical system, as elements of the former or as terms of the latter, and grammaticality of sentences is to be checked by computations on strings of types.

Two such categorial grammars are still being actively investigated. One has historical antecedents going back to Husserl, Lesniewski, Ajdukiewicz and Bar-Hillel. Its underlying formal system was transformed by the present author into what algebraists would call a "residuated monoid"[1)], or what he called the "syntactic calculus" [1958], a form of positive intuitionistic propositional logic, but without Gentzen's three *structural rules*: contraction, weakening and interchange (see Kleene [1952]). Refinements and generalizations of this system are still being pursued in articles and books by Moortgat, Oehrle, Morrill, Carpenter, Fadda, Stabler and others, most recently under the name of "type logical grammar" (see Moortgat [1997]).

A newer kind of categorial grammar was inspired by Claudia Casadio's [2001] proposal to replace the syntactic calculus by classical non-commutative linear logic (see also Casadio and Lambek [2002]). In retrospect, it turned out that our new approach had been anticipated by certain ideas of Zellig Harris [1966].[2)] It was elaborated into a partially ordered algebraic system, a *pregroup*, or, equivalently, into a logical system, *compact bilinear logic*. This new approach differed from the older one in having replaced two binary operations of *division* (binary connectives of *implication*) by two unary operations of *adjunction* (unary connectives of *negation*). Moreover, the new approach replaces the two-dimensional proof-trees of type logical grammars by one-dimensional calculations, resembling those used by mathematicians when simplifying algebraic expressions, and – it is hoped – mimicking what goes on in the minds of human speakers and hearers.

Anticipating both these approaches, Charles Sanders Peirce [1897] had pointed out that English words may require complements, which resemble "unsaturated bonds" or "valences" in Chemistry. For example, the transitive verb-form *sees* requires two complements, a third person subject on the left and a direct object on the right. We may illustrate Peirce's idea by showing how the same short sentence might be analyzed in the syntactic calculus as[3)]

(1.1) *he sees her*

$$\pi_3\ (\pi_3\backslash \mathbf{s}_1/\mathbf{o})\ \mathbf{o} \quad \rightarrow \mathbf{s}_1$$

and by Harris as

(1.2)
$$\begin{array}{c} \textit{he see her} \\ \pi_3\ (\pi_3^r\mathbf{s}_1\mathbf{o}^\ell)\mathbf{o} \quad \to \mathbf{s}_1 \end{array}$$

To facilitate comparison with my present approach I have adopted my present notation for basic types:

π_3 = third person singular subject,

$\mathbf{o}$ = direct object

$\mathbf{s}_1$ = declarative sentence in present tense.

We will assume here and henceforth that, for all basic types a,

$$a^\ell a \to 1, \quad aa^r \to 1.$$

A calculation then shows that

$$\pi_3(\pi_3^r\mathbf{s}_1\mathbf{o}^\ell)\mathbf{o} = (\pi_3\pi_3^r)\mathbf{s}_1(\mathbf{o}^\ell\mathbf{o}) \to 1\mathbf{s}_1 1 \to \mathbf{s}_1.$$

2. Recent developments.

Harris did not point out that the operation $a \mapsto a^\ell$ can be iterated, helping to describe Chomsky's "traces". To explain this new idea, let us first look at

(2.1)
$$\begin{array}{c} \textit{he will see her} \\ \pi_3\ (\pi^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{i}\mathbf{o}^\ell)\mathbf{o} \to \mathbf{s}_1. \end{array}$$

Here we have adopted the basic types

$\mathbf{j}$ = infinitive of intransitive verb phrase

$\mathbf{i}$ = same for non-auxiliary verbs

π = subject if the person does not matter

and we postulate

$$\pi_3 \to \pi, \quad \mathbf{i} \to \mathbf{j}.$$

The reason for distinguishing $\mathbf{j}$ from $\mathbf{i}$ will appear later. The calculation goes as follows:

$$\begin{array}{l} \pi_3(\pi^r\mathbf{s}_1\mathbf{j}^\ell) \to \pi\,\pi^r\mathbf{s}_1\mathbf{j}^\ell \to \mathbf{s}_1\mathbf{j}^\ell \\ (\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{i}\mathbf{o}^\ell) \to \mathbf{s}_1\mathbf{j}^\ell\mathbf{j}\mathbf{o}^\ell \to \mathbf{s}_1\mathbf{o}^\ell \\ (\mathbf{s}_1\mathbf{o}^\ell)\mathbf{o} \to \mathbf{s}_1. \end{array}$$

At least, this is how a speaker or hearer might proceed in stages. The grammarian may present this calculation in abbreviated form:

$$\pi_3(\pi^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{i}\mathbf{o}^\ell)\mathbf{o} \ \to \mathbf{s}_1\ .$$

Next consider the question

(2.2)
$$\begin{array}{c} \textit{will he see her ?} \\ (\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3\ (\mathbf{i}\mathbf{o}^\ell)\ \mathbf{o} \quad \to \mathbf{q}_1. \end{array}$$

Here

$$\mathbf{q}_1 = \text{ yes-or-no question in present tense}$$

and *will* has been assigned a second type $\mathbf{q}_1\mathbf{j}^\ell\pi^\ell$. Instead of expecting the dictionary to list both types $\pi^r\mathbf{s}_1\mathbf{j}^\ell$ and $\mathbf{q}_1\pi^\ell\mathbf{j}^\ell$ for *will*, we adopt the metarule (apologies to Gazdar):

I. If a modal or auxiliary verb has type $\pi_k^r\mathbf{s}_j\mathbf{j}^\ell$, then it also has type $\mathbf{q}_j\pi_k^\ell\mathbf{j}^\ell$, and similarly with π_k replaced by π or $\mathbf{j}$ replaced by $\mathbf{i}$.

Here $k = 1, 2, 3$ stands for the first, second and third person respectively, and $j = 1, 2$ stands for the present and past tense respectively. We have ignored the two subjunctive tenses, which are almost obsolete, and we let π_2, the type of *you*, also stand for all three persons of the plural in English[4)] It may be pointed out that, in German, Metarule I would apply to all verbs, not just to modal and auxiliary ones.

From now on, we will assume that the set of basic types is endowed with a partial order,[5)] here denoted by an arrow, namely a transitive, reflexive and antisymmetric relation. In particular, we have adopted the postulates

$$\pi_k \to \pi, \quad \mathbf{q}_3 \to \mathbf{q}$$

where

$$\mathbf{q} = \text{ yes-or-no question when the tense does not matter.}$$

Next, look at

$$\begin{array}{c} \textit{whom will he see} - ? \\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3\ (\mathbf{i}\mathbf{o}^\ell) \to \overline{\mathbf{q}} \end{array} \tag{2.3}$$

Here a new basic type makes its appearance:

$$\overline{\mathbf{q}} = \text{ question,}$$

including not only yes-or-no questions, but also wh-questions. We postulate

$$\mathbf{q}_j \ \to \ \mathbf{q} \ \to \ \overline{\mathbf{q}}.$$

It is necessary to maintain the distinction between $\mathbf{q}$ and $\overline{\mathbf{q}}$, since e.g. (2.2) can be preceded by *when* of type $\overline{\mathbf{q}}\mathbf{q}^\ell$, but (2.3) cannot. The reason why we have used $\mathbf{q}$ instead of $\mathbf{q}_1$ in the type of *whom* is that this word is independent of tense.

The underlinks in (2.1) to (2.3) go back to Harris [1966]. They may be viewed as degenerate instances of what linear logicians call "proofnets". From a linguist's point of view, they represent a "deep structure" (apologies to Chomsky), which may also be indicated by square brackets, for instance in (2.3):

$$\overline{\mathbf{q}}[\mathbf{o}^{\ell\ell}[\mathbf{q}^\ell\mathbf{q}_1][\mathbf{j}^\ell[\pi^\ell\pi_3]\mathbf{i}]\mathbf{o}^\ell]$$

Note that this differs from the "surface structure" indicated by ordinary parentheses in (2.3). Because of the difficulty in producing nested underlinks, we will avoid them from now on by breaking up the calculations of (2.2) and (2.3) into separate steps:

$$\begin{array}{c} \mathbf{q}_1\mathbf{j}^\ell\pi^\ell\pi_3\mathbf{i}\mathbf{o}^\ell\mathbf{o} \to \mathbf{q}_1\mathbf{j}^\ell\mathbf{i} \to \mathbf{q}_1\ , \\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^\ell\mathbf{i}\mathbf{o}^\ell \to \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^\ell \to \overline{\mathbf{q}}, \end{array}$$

with the help of (generalized) contractions

$$\pi^\ell \pi_3 \to \pi^\ell \pi \to 1,\ \mathbf{o}^\ell \mathbf{o} \to 1,\ \mathbf{j}^\ell \mathbf{i} \to \mathbf{j}^\ell \mathbf{j} \to 1.$$

My authority for saying *whom* rather than *who* is the late Inspector Morse (see Dexter [1994]), who kept on reminding his sergeant: "*whom, Lewis, whom*". However, not only Sergeant Lewis, but even Noam Chomsky and Steven Pinker accept *who* as the natural usage for the object pronoun. Pinker (p.110) asserts: "In the U.S. *whom* is used consistently only by careful writers and pretentious speakers." I apologize for being a pretentious speaker; but, English being my second language, the object pronoun *whom* comes to me more naturally than *who*.

The final dash after *see* in (2.3) represents a Chomskyan trace. It turns out that double adjoints, not considered by Harris, occur wherever modern European languages would require traces. Double adjoints are also useful for typing clitic pronouns in Romance languages (see e.g. Bargelli and Lambek [2001]). Double adjoints have not yet shown up in preliminary investigations of Latin, Turkish and Arabic. A triple adjoint first turned up during the preparation of the present article. (See (2.6) below.)

Consider the question

(2.4)
$$\begin{array}{l} \textit{will he go with her ?} \\ (\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3[\mathbf{i}(\mathbf{j}^r\mathbf{i}\mathbf{o}^\ell)\mathbf{o} \end{array} \quad \to \quad \mathbf{q}_1\mathbf{j}^\ell[\mathbf{i}\mathbf{j}^r\mathbf{i} \to \mathbf{q}_1\mathbf{j}^\ell\mathbf{i} \to \mathbf{q}_1.$$

Here the preposition *with* has been assigned the type $\mathbf{j}^r\mathbf{i}\mathbf{o}^\ell$ to ensure that *go with* behaves like a transitive verb of type $\mathbf{i}\mathbf{o}^\ell$. We have retained a single square bracket in front of the first $\mathbf{i}$ in (2.4); it serves as a kind of punctuation mark to remind us that the question does not end with *go* and the tempting contraction $\mathbf{j}^\ell\mathbf{i} \to 1$ must be postponed.

We are now led to the wh-question

(2.5)
$$\begin{array}{l} \textit{whom will he go with } - \textit{ ?} \\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3[\mathbf{i}\ (\mathbf{j}^r\mathbf{i}\mathbf{o}^\ell) \end{array} \quad \to \quad \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^\ell\mathbf{i}\mathbf{o}^\ell \to \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^\ell \to \overline{\mathbf{q}}.$$

Prescriptive grammarians tell us that a preposition is something with which we are not supposed to end a sentence. Following their advice, we may reformulate (2.5) thus:

(2.6)
$$\begin{array}{l} \textit{with whom will he go ?} \\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\overline{\mathbf{q}}^\ell)(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell)\ \mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\ \pi_3\mathbf{i} \end{array} \quad \to \quad \overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\mathbf{o}^{\ell\ell}\mathbf{j}^\ell\mathbf{i} \to \overline{\mathbf{q}}.$$

where we have assigned the new type $\overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\overline{\mathbf{q}}^\ell$ to the preposition *with*. We note that *with whom* then has the same type $\overline{\mathbf{q}}\mathbf{q}^\ell$ as *when*.

Presumably, Sergeant Lewis would not be happy with the analysis (2.6), since he had assigned the type $\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell$ to *who*, and even he would not say *with who*. Here is one way he might have analyzed the same sentence:

(2.7)
$$\begin{array}{l} \textit{with whom will he go } - \textit{ ?} \\ (\mathbf{j}^r\mathbf{i}\mathbf{o}^\ell)(\mathbf{o}\mathbf{i}^r\mathbf{j}^{rr}\overline{\mathbf{q}}\mathbf{q}^\ell)(\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3(\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3\mathbf{i} \end{array} \quad \begin{array}{l} \to \mathbf{j}^r\mathbf{i}\mathbf{i}^r\mathbf{j}^{rr}\overline{\mathbf{q}}\mathbf{j}^\ell\mathbf{i} \\ \to \mathbf{j}^r\mathbf{j}^{rr}\overline{\mathbf{q}} \to \overline{\mathbf{q}} \end{array}$$

He would thus preserve the old type of *with* and avoid triple adjoints, at the cost of introducing a more complicated new type for *whom*, which reflects his dislike of this word-form. However, assuming that adverbs can also have type $\mathbf{s}\mathbf{s}^\ell$ he might have assigned to the preposition *with* the additional type $\mathbf{s}\mathbf{s}^\ell\mathbf{o}^\ell$, hence to *whom* the type $\mathbf{o}\mathbf{s}\mathbf{s}^r\overline{\mathbf{q}}\mathbf{q}^\ell$.

3. Pregroups.

The time has come to describe our formal system. A *pregroup* is a partially ordered monoid[5)] in which each element a has a *left adjoint* a^ℓ and a *right adjoint* a^r satisfying

$$a^\ell a \to 1 \to aa^\ell,\ aa^r \to 1 \to a^r a\ .$$

It is easily shown that

(3.1) adjoints are unique;

(3.2) $1^\ell = 1 = 1^r,\ a^{r\ell} = a = a^{\ell r}$;

$(ab)^\ell = b^\ell a^\ell,\ (ab)^r = b^r a^r$;

(3.3) if $a \to b$ then $b^\ell \to a^\ell$ and $b^r \to a^r$.

Our first step in approaching the grammar of a natural language is to assign to each word a *type*, namely a string of *simple types* of the form

$$\cdots\ a^{\ell\ell},\ a^\ell,\ a,\ a^r,\ a^{rr},\ \cdots$$

where a is any *basic type*, an element of a given partially ordered set. This set is assumed to have been chosen to represent certain fundamental grammatical entities (categories) and their features. Mathematically speaking, this amounts to working in the pregroup *freely generated* by the partially ordered set of basic types.

Certain postulates, such as $\pi_k \to \pi$ and $\mathbf{q}_j \to \mathbf{q}$, may be incorporated into this given partial order. We are not permitted to introduce postulates of the form $\alpha \to \beta$ when α or β is not basic, for then the pregroup of types would no longer be free.

Why do we insist on *free* pregroups? The reader will have noticed that the examples in Section 2 involve only *contractions* $a^\ell a \to 1$ and $aa^r \to 1$ and not *expansions* $1 \to aa^\ell$ and $1 \to a^r a$. The reason for this is the following

Switching Lemma. Without loss of generality, one may assume that, in any calculation of $\alpha \to \beta$ in the free pregroup generated by a partially ordered set, *generalized contractions*

$$b^\ell a \to b^\ell b \to 1,\ ab^r \to bb^r \to 1,$$

assuming that $a \to b$, precede *generalized expansions*

$$1 \to aa^\ell \to ba^\ell,\ 1 \to a^r a \to a^r b.$$

For a formal proof of this see Lambek [1999]; but the following observation will give an idea of the proof.

Suppose $a \to b$ and $b \to c$, then

$$a = a1 \to ab^r b \to ab^r c \to bb^r c \to 1c = c$$

can be replaced by

$$a \to b \to c$$

using just transitivity of the arrow. Hence a generalized expansion such as

$$1 \to b^r b \to b^r c$$

need not immediately precede a generalized contraction such as

$$ab^r \to bb^r \to 1.$$

Note that, in a free pregroup, all calculations can involve only (generalized) contractions and expansions in addition to the postulates that were incorporated into the partially ordered set of basic types.

As a corollary to the Switching Lemma, we note that, when $\beta = b$ is a *simple* type, the proof of $\alpha \to \beta$ need not involve any expansions at all. In all our examples, the element on the right hand side was $\mathbf{s}_j, \mathbf{s}, \mathbf{q}_j, \mathbf{q}$ or $\overline{\mathbf{q}}$, as required for verification of sentencehood. However, expansions are needed to prove the equations (3.2) and the contravariance (3.3).[6)]

4. The English verb.

Since verbs are an essential ingredient of sentences, we cannot investigate the latter without first looking at the construction of verb-forms, which is usually called "conjugation". In some European languages, this is a major part of the grammar. In English it plays only a minor rôle, but one that should not be neglected, even if some texts on transformational grammar manage to do so.

The English verb has four simple tenses; but here we will consider only the present and past of the so-called "indicative mood" and ignore the almost obsolete subjunctive. While most European languages require six persons, three will suffice for the English verb, since the modern second person singular always yields the same verb-form as the three persons of the plural. Thus, with each verb V, we may associate a 2×3 matrix of so-called "finite" forms $C_{jk}V$, as illustrated by

$$C\ be \to \begin{pmatrix} am\ are\ is \\ was\ were\ was \end{pmatrix},$$

$$C\ go \to \begin{pmatrix} go\ go\ goes \\ went\ went\ went \end{pmatrix},$$

$$C\ will \to \begin{pmatrix} will\ will\ will \\ would\ would\ would \end{pmatrix}.$$

It is convenient to regard *would* formally as the past tense of *will*, as is justified historically, even if not semantically.

In addition to the infinitives, such as *be* and *go* (but not *will*) and to the finite forms, as above, there are two participles P_jV, as illustrated by

$$P\ be \to \begin{pmatrix} being \\ been \end{pmatrix},$$

$$P\ go \to \begin{pmatrix} going \\ gone \end{pmatrix},$$

English modals have neither infinitives nor participles. They have only the finite forms of type $\pi^r \mathbf{s}_j \mathbf{j}^\ell$, as we saw for *will* in Section 2. Some auxiliary verbs other than modals require special consideration, inasmuch as they may lack certain forms.

The perfect auxiliary *have* of type $\mathbf{j}\mathbf{p}_2^\ell$ has no participles;[7] we can say

(4.1)
$$\begin{array}{c} \textit{I may have gone} \\ \pi_1\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)\ \mathbf{p}_2 \end{array} \quad \rightarrow\ \mathbf{s}_1$$

but not

I may have **had gone*, *I may be* **having gone*.

The progressive auxiliary *be* of type $\mathbf{j}\mathbf{p}_1^\ell$ has no present participle[8]; we can say

(4.2)
$$\begin{array}{c} \textit{I may be going} \\ \pi_1\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_1^\ell)\ \mathbf{p}_1 \end{array} \quad \rightarrow \mathbf{s}_1$$

and

(4.3)
$$\begin{array}{c} \textit{I may have been going} \\ \pi_1\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)\ (\mathbf{p}_2 \mathbf{p}_1^\ell)\ \mathbf{p}_1 \end{array} \quad \rightarrow \mathbf{s}_1$$

but not

I may be **being going* .

The reader will have noticed that, when V is an intransitive verb such as *go*, we have assigned to $P_j V$ the type $\mathbf{p}_j$.

The passive auxiliary *be* of type $\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell$ does not lack any forms. We can say

(4.4)
$$\begin{array}{c} \textit{I may be seen} - \\ \pi_1\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{o}^\ell) \end{array} \quad \rightarrow\ \mathbf{s}_1 \mathbf{o}^{\ell\ell}\mathbf{o}^\ell \rightarrow \mathbf{s}_1$$

(4.5)
$$\begin{array}{c} \textit{I may have been seen} - \\ \pi_1\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{o}^\ell) \end{array} \quad \rightarrow\ \mathbf{s}_1 \mathbf{o}^{\ell\ell}\mathbf{o}^\ell \rightarrow \mathbf{s}_1$$

With some hesitation, we can even say

(4.6)
$$\begin{array}{c} \textit{I may be being seen} - \\ \pi_1\ (\pi^2 \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{o}^\ell) \end{array} \quad \rightarrow\ \mathbf{s}_1 \mathbf{o}^{\ell\ell}\mathbf{o}^\ell \rightarrow \mathbf{s}_1$$

and

(4.7)
$$\begin{array}{c} \textit{I may have been being seen} - \\ \pi_1\ (\pi \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{o}^\ell) \end{array} \quad \rightarrow\ \mathbf{s}_1 \mathbf{o}^{\ell\ell}\mathbf{o}^\ell \rightarrow \mathbf{s}_1$$

The emphatic auxiliary *do* of type $\mathbf{j}\mathbf{i}^\ell$ (also used for questions and negation) has only finite forms. We can say

(4.8)
$$\begin{array}{c} \textit{I do go} \\ \pi_1\ (\pi_1^r \mathbf{s}_1 \mathbf{i}^\ell)\ \mathbf{i} \end{array} \ \rightarrow\ \mathbf{s}_1$$

but not

*I have *done go, I am *doing go, I may *do go.*

The reason why all the infinitives of auxiliary verbs were assigned a type of the form $\mathbf{j}x^\ell$ rather than $\mathbf{i}x^\ell$ is that they cannot be preceded by the auxiliary *do*. We do not normally say

$$\begin{array}{c} {}^*\ \textit{I do be going} \\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{i}^\ell)(\mathbf{j}\mathbf{p}_1^\ell)\mathbf{p}_1 \quad \not\to \quad \mathbf{s}_1 \end{array}$$

$$\begin{array}{c} {}^*\ \textit{I do have gone} \\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{i}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)\mathbf{p}_2 \quad \not\to \quad \mathbf{s}_1 \end{array}$$

$$\begin{array}{c} {}^*\ \textit{I do be seen} \\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{i}^\ell)(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)\mathbf{p}_2 \quad \not\to \quad \mathbf{s}_1 \end{array}$$

Among *main* verbs, that is verbs which are not auxiliary, we distinguish infinitives of intransitive verbs of type $\mathbf{i}$, of transitive verbs of type $\mathbf{io}^\ell$ and of verbs with more than one complement, say of type $\mathbf{i}y^\ell\mathbf{o}^\ell$. The last will be considered in Sections 9 and 10. It is often convenient to treat finite forms of these verbs by assigning separate types to the *inflectors* and to the infinitives, and the same goes for the passive auxiliary *be*, whose infinitive has type $\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell$. The inflectors are

$$C_{jk} \text{ of type } \pi_k^r\mathbf{s}_j j^\ell, \qquad P_j \text{ of type } \mathbf{p}_j\mathbf{j}^\ell.$$

For example, we may analyze *sees* thus:

$$\begin{array}{c} C_{31}\ \textit{see} \ \to\ \textit{sees} \\ (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{io}^\ell) \ \to\ {\pi_3}^r\mathbf{s}_1\mathbf{o}^\ell \end{array}$$

We may think of the upper arrow as living in the production grammar which generates the verb form *sees*, in contrast to the lower arrow, which represents the partial order in the free pregroup of types. By assigning the unreduced type $\pi_3^r\mathbf{s}_1\mathbf{j}^\ell[\mathbf{io}^\ell$ to *sees*, we can calculate

$$\text{(4.7)} \qquad \begin{array}{c} \textit{he sees her tomorrow} \\ \pi_3\ (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell[\mathbf{io}^\ell)\ \mathbf{o}\ (\mathbf{j}^r\mathbf{i}) \end{array} \ \to\ \mathbf{s}_1\mathbf{j}^\ell[\mathbf{i}\,\mathbf{j}^r\mathbf{i} \to \mathbf{s}_1\mathbf{j}^\ell\ \mathbf{i} \to \mathbf{s}_1.$$

The left square bracket here serves as a warning not to contract $\mathbf{j}^\ell\mathbf{i} \to 1$ prematurely, lest the sentence stop after *her*.

Similarly, analyzing

$$\begin{array}{c} P_1\ \textit{see} \ \to\ \textit{seeing} \\ (p_1\mathbf{j}^\ell)(\mathbf{io}^\ell) \to \mathbf{p}_1\mathbf{o}^\ell \end{array}$$

we can justify

$$\text{(4.8)} \qquad \begin{array}{c} \textit{he is seeing her tomorrow} \\ \pi_3(\pi_3^r\mathbf{s}_1\mathbf{p}_1^\ell)(\mathbf{p}_1\mathbf{j}^\ell[\mathbf{io}^\ell)\mathbf{o}(\mathbf{j}^r\mathbf{i}) \end{array} \ \to\ \mathbf{s}_1\mathbf{j}^\ell[\mathbf{i}\mathbf{j}^r\mathbf{i} \to \mathbf{s}_1\mathbf{j}^\ell\mathbf{i} \to \mathbf{s}_1$$

and analyzing the passive auxiliary

$$\begin{array}{c} C_{13}\ \textit{be} \ \to\ \textit{is} \\ (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell) \to \pi_3^r\mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell \end{array}$$

we justify

$$\text{(4.9)} \qquad \begin{array}{l} \textit{he is seen} \ - \ \textit{tomorrow} \\ \pi_3\ (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell [\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)\ (\mathbf{p}_2\mathbf{o}^\ell)(\mathbf{j}^r\mathbf{i}) \ \rightarrow\ \mathbf{j}^\ell[\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{o}^\ell\mathbf{j}^r\mathbf{i} \\ \qquad\qquad \rightarrow \mathbf{s}_1\mathbf{j}^\ell[\mathbf{j}\mathbf{j}^r\mathbf{i} \ \rightarrow\ \mathbf{s}_1\mathbf{j}^\ell\mathbf{i} \rightarrow \mathbf{s}_1 \end{array}$$

In the above examples, the adverb *tomorrow* may be replaced by a prepositional phrase such as

$$\begin{array}{l} \textit{on Tuesday} \\ (\mathbf{j}^r\mathbf{i}\mathbf{o}^\ell)\ \mathbf{o} \quad \rightarrow\ \mathbf{j}^r\mathbf{i} \end{array}$$

or by a subordinate clause such as

$$\begin{array}{l} \textit{when it rains} \\ (\mathbf{j}^r\mathbf{i}\mathbf{s}^\ell)\pi_3\ (\pi_3^r\mathbf{s}_1) \ \rightarrow\ \mathbf{j}^r\mathbf{i}\mathbf{s}^\ell\mathbf{s}_1 \rightarrow \mathbf{j}^r\mathbf{i}\ . \end{array}$$

5. The object pronoun *whom.*

To give a brief account of the new algebraic approach to syntax, I have decided to focus here on the object pronoun *whom*, which may serve to introduce questions or relative clauses, at least according to Inspector Morse. Not surprisingly, different contexts require that it be assigned different types. In fact, according to its function, the English question-word *whom* translates into different German words:

wen (asking for direct object),

wem (asking for indirect object),

and the English relative pronoun *whom* translates into German:

den (masculine accusative),

die (feminine or plural accusative),

das (neuter accusative),

dem (masculine or neuter dative),

der (feminine dative),

denen (plural dative).

For typing the English object pronoun *whom*, gender and number are irrelevant, but the case distinction between direct and indirect object, though invisible morphologically, may be significant syntactically. Followers of Sergeant Lewis would also abolish the morphological distinction between subject and object, except perhaps for objects of prepositions.

Of course, *whom* also embodies the feature "human"; but this belongs to semantics, not to syntax. Consider, for example,

I saw the mouse whom the cat ate −

This is a well-formed grammatical sentence, but it anthropomorphically ascribes humanity to the mouse. As a rule, when referring to non-humans, *whom* should be replaced by *which* in relative clauses and by *what* in questions. In relative clauses, it may also be replaced by *that*, or even by $\emptyset$, but more about this later (see Section 11).

6. *Whom* **in questions**.

When I first proposed pregroup grammars in 1998, Michael Moortgat asked how one would handle a wh-question such as (2.3) when it is followed by an adverb such as *tomorrow*? My present answer is to assign a new type to *see*, refining the old type $\mathbf{i}\mathbf{o}^\ell$ to $\mathbf{i}\mathbf{o}^\ell\mathbf{j}^\ell\mathbf{i} \to \mathbf{i}\mathbf{o}^\ell$. Thus we have

(6.1)
$$\begin{array}{l} \textit{whom will he see} \ - \ \textit{tomorrow}\ ? \\ (\bar{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_3(\mathbf{i}\mathbf{o}^\ell\mathbf{j}^\ell[\mathbf{i})(\mathbf{j}^r\mathbf{i}) \\ \to\ \bar{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^\ell\,\mathbf{i}\mathbf{o}^\ell\mathbf{j}^\ell\mathbf{i} \to \bar{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^\ell \to \bar{\mathbf{q}} \end{array}$$

Here the adverb *tomorrow* can be replaced by any prepositional phrase or subordinate clause of the same type, such as *on Tuesday* or *when it rains.*

Rather than list the new type for *see* in the dictionary, we adopt the following metarule:

II. The type $\mathbf{i}\mathbf{o}^\ell$ of any transitive verb may be refined to

$$\mathbf{i}\mathbf{o}^\ell\mathbf{j}^\ell\mathbf{i} \ \to\ \mathbf{i}\mathbf{o}^\ell.$$

This metarule applies not only when *whom* introduces direct questions, but also when it introduces indirect questions, as we shall now see, or relative clauses, as we shall see in Section 8. Here are two indirect questions:

(6.2)
$$\begin{array}{l} \textit{I wonder whom I will see tomorrow} \\ \pi_1\ (\pi_1^r\mathbf{s}_1\mathbf{r}^\ell)(\mathbf{r}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell)\pi_1\ (\pi^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{i}\mathbf{o}^\ell\mathbf{j}^\ell[\mathbf{i})(\mathbf{j}^r\mathbf{i}) \\ \to\ \mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{s}^\ell\mathbf{s}_1\mathbf{o}^\ell\mathbf{j}^\ell\mathbf{i} \to \mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{o}^\ell \to \mathbf{s}_1. \end{array}$$

Here

$$\mathbf{r} = \ \text{indirect question},$$

and *wonder* has the type

$$(\pi_1^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{i}\mathbf{r}^\ell) \ \to\ \pi_1^r\mathbf{s}_1\mathbf{r}^\ell$$

(6.3)
$$\begin{array}{l} \textit{I wonder whom to see tomorrow} \\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{r}^\ell)(\mathbf{r}\mathbf{o}^{\ell\ell}\bar{\mathbf{j}}^\ell)(\bar{\mathbf{j}}\mathbf{j}^\ell)(\mathbf{i}\mathbf{o}^\ell\mathbf{j}^\ell[\mathbf{i})(\mathbf{j}^r\mathbf{i}) \\ \to\ \mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{o}^\ell\mathbf{j}^\ell\mathbf{i} \to \mathbf{s}_1 \end{array}$$

Here

$\bar{\mathbf{j}}$ = complete infinitive (with *to*)

so *to* has type $\bar{\mathbf{j}}\mathbf{j}^{\ell}$.

7. ***Whom* as a relative pronoun**.

In addition to serving as a question word, *whom* may also be a relative pronoun, as in a restrictive relative clause, such as

$$\textit{the girl}(s)\ \textit{whom I will see}\ -\ \textit{tomorrow}$$

or in a non-restrictive one, such as

$$\textit{the girl}(s),\ \textit{whom I will see}\ -\ \textit{tomorrow}\ .$$

We can handle the noun phrases by assigning to *whom* the new type

$$x^r x \mathbf{o}^{\ell\ell}\mathbf{s}^{\ell},$$

where $x = \mathbf{c},\ \mathbf{p},\ \overline{\mathbf{c}}$ or $\overline{\mathbf{p}}$. Here

$\mathbf{c}$ = count noun, such as *pig*,

$\mathbf{p}$ = plural, such as *pigs*,

$\overline{\mathbf{c}}$ = complete singular noun phrase, such as *a pig*,

$\overline{\mathbf{p}}$ = complete plural, such as *many pigs*.

We must postulate

$$\overline{\mathbf{c}} \to \pi_3,\ \mathbf{o};\quad \mathbf{p} \to \overline{\mathbf{p}} \to \pi_2,\ \mathbf{o}$$

to ensure that complete noun phrases may occur in subject or object position and that plural nouns do not require a determiner. There are also mass nouns, say of type $\mathbf{m}$, such as *pork*, but these normally require the relative pronoun *which* in place of *whom* on semantic grounds. The following examples illustrate the cases $x = \mathbf{c}$ and $x = \overline{\mathbf{c}}$ respectively:

(7.1)
$$\begin{array}{c} \textit{the girl whom I will see}\ - \\ (\overline{\mathbf{c}}\mathbf{c}^{\ell})[\mathbf{c}(\mathbf{c}^r\mathbf{c}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\pi_1\ (\pi^r\mathbf{s}_1\mathbf{j}^{\ell})\ (\mathbf{i}\mathbf{o}^{\ell}) \\ \to\ \overline{\mathbf{c}}\mathbf{c}^{\ell}\mathbf{c}\ \mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\mathbf{s}_1\mathbf{o}^{\ell} \to \overline{\mathbf{c}}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell} \to \overline{\mathbf{c}} \end{array}$$

(7.2)
$$\begin{array}{c} \textit{a girl, whom I will see}\ - \\ (\overline{\mathbf{c}}\mathbf{c}^{\ell})\ \mathbf{c}(\overline{\mathbf{c}}^r\overline{\mathbf{c}}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\pi_1\ (\pi^r\mathbf{s}_1\mathbf{j}^{\ell})\ (\mathbf{i}\mathbf{o}^{\ell}) \\ \to\ \overline{\mathbf{c}}\overline{\mathbf{c}}^r\overline{\mathbf{c}}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\mathbf{s}_1\mathbf{o}^{\ell} \to \overline{\mathbf{c}}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell} \to \overline{\mathbf{c}} \end{array}$$

If the relative clauses are followed by an adverb such as *tomorrow*, the type of *see* must be refined according to Metarule II. In addition to the restrictive and non-restrictive relative clauses illustrated by (7.1) and (7.2) respectively, McCawley discusses also other kinds: cleft, pseudo-relative, free relative and infinitival relative clauses, etc. To keep this article in reasonable bounds, we will ignore these.

8. *Whom* **with a preposition**.

Let us return once more to the question raised in Section 2: what happens when *whom* is combined with a preposition? Consider the following three examples:

(8.1)
$$\begin{gathered}\textit{whom will she go with tomorrow ?}\\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_3[\mathbf{i}\ (\mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}[\mathbf{i})\ (\mathbf{j}^r\mathbf{i})\\ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}\mathbf{i} \to \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell} \to \overline{\mathbf{q}}\end{gathered}$$

(8.2)
$$\begin{gathered}\textit{I wonder whom she will go with tomorrow}\\ \pi_1\ (\pi_1^r\mathbf{s}_1\mathbf{r}^{\ell})(\mathbf{r}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\pi_3(\pi^r\mathbf{s}_1\mathbf{j}^{\ell})\ [\mathbf{i}(\mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}[\mathbf{i})(\mathbf{j}^r\mathbf{i})\\ \to\ \mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\mathbf{s}_1\mathbf{j}^{\ell}\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}\mathbf{i} \to \mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell} \to \mathbf{s}_1\end{gathered}$$

(8.3)
$$\begin{gathered}\textit{boys whom she will go with tomorrow}\\ \mathbf{p}\ (\mathbf{p}^r\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\pi_3\ (\pi^r\mathbf{s}_1\mathbf{j}^{\ell})\ [\mathbf{i}(\mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}[\mathbf{i})(\mathbf{j}^r\mathbf{i})\\ \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\mathbf{s}_1\mathbf{j}^{\ell}\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}\mathbf{i} \to \mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell} \to \mathbf{p}\end{gathered}$$

In all three examples, the preposition *with* should be assigned the type

$$\mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell}\mathbf{j}^{\ell}\mathbf{i} \to \mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell},$$

although in the presence of the adverb *tomorrow* the contraction $\mathbf{j}^{\ell}\mathbf{i} \to 1$ must be blocked. Prescriptive grammarians would replace these as follows:

(8.4)
$$\begin{gathered}\textit{with whom (will she go tomorrow) ?}\\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\overline{\mathbf{q}}^{\ell})(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})\ \mathbf{q}_1\ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\mathbf{o}^{\ell\ell} \to \overline{\mathbf{q}}\end{gathered}$$

(8.5)
$$\begin{gathered}\textit{I wonder with whom (she will go tomorrow) ?}\\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{r}^{\ell})(\mathbf{r}\mathbf{o}^{\ell\ell\ell}\mathbf{r}^{\ell})\ \mathbf{r}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\mathbf{s}\ \to\ \mathbf{s}_1\mathbf{o}^{\ell\ell\ell}\mathbf{o}^{\ell\ell} \to \mathbf{s}_1\end{gathered}$$

(8.6)
$$\begin{gathered}\textit{boys with whom (she will go tomorrow)}\\ \mathbf{p}(\mathbf{p}^r\mathbf{p}\mathbf{o}^{\ell\ell\ell}\mathbf{p}^{\ell}[\mathbf{p})(\mathbf{p}^r\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\ \mathbf{s}_1\\ \to\ \mathbf{p}\mathbf{o}^{\ell\ell\ell}\mathbf{p}^{\ell}\mathbf{p}\mathbf{o}^{\ell\ell}\ \to\ \mathbf{p}\mathbf{o}^{\ell\ell\ell}\mathbf{o}^{\ell\ell} \to \mathbf{p}\end{gathered}$$

To explain the new types for the preposition *with* in the last three examples, we adopt the metarule:

III. A preposition of type $\mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell}$ also has type $y\mathbf{o}^{\ell\ell\ell}y^{\ell}$, where $y = \mathbf{q}, \mathbf{r}$ or $x^r x$ and $x = \mathbf{c}, \overline{\mathbf{c}}, \mathbf{p}$ or $\overline{\mathbf{p}}$.

Note that

$$(\mathbf{p}^r\mathbf{p})^{\ell} = \mathbf{p}^{\ell}\mathbf{p}^{r\ell} = \mathbf{p}^{\ell}\mathbf{p}.$$

Although (8.4) to (8.6) involve a triple adjoint, otherwise their analysis appears to be simpler than that of (8.1) to (8.3). In some examples, one is definitely tempted to side with the prescriptive grammarians. For instance, compare the following two alternatives:

(8.7)
$$\begin{gathered}\textit{whom did you bring that book which I do not}\\ \textit{want to be read to from for ?}\end{gathered}$$

$$(8.8)\qquad \begin{gathered}\textit{for whom did you bring that book from which}\\ \textit{I do not want to be read to ?}\end{gathered}$$

It seems easier to parse the supposedly pedantic (8.8) than the allegedly more natural (8.7).

Note that Sergeant Lewis might analyze (8.6) differently:

$$(8.9)\qquad \begin{gathered}\textit{boys with whom (she will go tomorrow)}\\ \mathbf{p}(\mathbf{s}\mathbf{s}^{\ell}\mathbf{o}^{\ell})(\mathbf{o}\mathbf{s}\mathbf{s}^{r}\mathbf{p}^{r}\mathbf{p}\mathbf{s}^{\ell})\mathbf{s}\\ \to\ \mathbf{p}\mathbf{s}\mathbf{s}^{\ell}\mathbf{s}\mathbf{s}^{r}\mathbf{p}^{r}\mathbf{p}\to\mathbf{p}\mathbf{s}\mathbf{s}^{r}\mathbf{p}^{r}\mathbf{p}\to\mathbf{p}\mathbf{p}^{r}\mathbf{p}\to\mathbf{p}.\end{gathered}$$

9. Verbs with two complements.

Infinitives of verbs with two complements may have type $\mathbf{i}y^{\ell}\mathbf{o}^{\ell}$, with $y=\mathbf{j}$, $\bar{\mathbf{j}}$ or $\mathbf{r}$ for example, as in

$$\begin{gathered}\textit{let her go}\\ (\mathbf{i}\mathbf{j}^{\ell}\mathbf{o}^{\ell})\mathbf{o}\ \mathbf{i}\ \to\ \mathbf{i}\mathbf{j}^{\ell}\mathbf{i}\to\mathbf{i}\\[6pt] \textit{tell her to go}\\ (\mathbf{i}\bar{\mathbf{j}}^{\ell}\mathbf{o}^{\ell})\mathbf{o}(\bar{\mathbf{j}}\mathbf{j}^{\ell})\mathbf{i}\ \to\mathbf{i}\bar{\mathbf{j}}^{\ell}\bar{\mathbf{j}}\to\mathbf{i}\\[6pt] \textit{ask her when to go}\\ (\mathbf{i}\mathbf{r}^{\ell}\mathbf{o}^{\ell})\mathbf{o}(\mathbf{r}\bar{\mathbf{j}}^{\ell})(\bar{\mathbf{j}}\mathbf{j}^{\ell})\mathbf{i}\ \to\mathbf{i}\mathbf{r}^{\ell}\mathbf{r}\to\mathbf{i}\ .\end{gathered}$$

We recall that

$$\begin{aligned}\bar{\mathbf{j}} &= \text{ complete infinitive (with } \textit{to})\\ \mathbf{r} &= \text{ indirect question}\end{aligned}$$

A problem arises with verbs of type $\mathbf{i}y^{\ell}\mathbf{o}^{\ell}$, not only in connection with *whom*, but already with the passive construction. For example, in case $y=\bar{\mathbf{j}}$, how do we handle

$$\begin{gathered}\textit{he will be told to go}\\ \pi_3(\pi^{r}\mathbf{s}_1\mathbf{j}^{\ell})(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\bar{\mathbf{j}}^{\ell}\mathbf{o}^{\ell})(\bar{\mathbf{j}}\mathbf{j}^{\ell})\mathbf{i}\ \to\mathbf{s}_1\mathbf{o}^{\ell\ell}\bar{\mathbf{j}}^{\ell}\mathbf{o}^{\ell}\bar{\mathbf{j}}\to\ ?\end{gathered}$$

$$\begin{gathered}\textit{whom will I tell to go}\\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_1(\mathbf{i}\bar{\mathbf{j}}^{\ell}\mathbf{o}^{\ell})(\bar{\mathbf{j}}\mathbf{j}^{\ell})\mathbf{i}\ \to\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}\mathbf{i}\bar{\mathbf{j}}^{\ell}\mathbf{o}^{\ell}\bar{\mathbf{j}}\\ \to\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\bar{\mathbf{j}}^{\ell}\mathbf{o}^{\ell}\bar{\mathbf{j}}\to\ ?\end{gathered}$$

where the string of types does not contract as expected. One way to resolve this problem is to derive these sentences from the *pseudo-sentence*

$$\begin{gathered}{}^{*}\ \textit{I tell to go him}\\ \pi_1(\pi_1{}^{r}\mathbf{s}_1\hat{\mathbf{o}}^{\ell}\bar{\mathbf{j}}^{\ell})(\bar{\mathbf{j}}\mathbf{j}^{\ell})\mathbf{i}\ \mathbf{o}\ \to\ \mathbf{s}_1\hat{\mathbf{o}}^{\ell}\mathbf{o}\not\to\mathbf{s}_1\end{gathered}$$

This is not a sentence, as long as $\mathbf{o}\not\to\hat{\mathbf{o}}$, or as long as *him* does not have the type $\hat{\mathbf{o}}$,

$\hat{\mathbf{o}}$ = pseudo-object.

Thus, we assign to the verb in question the new type $\mathbf{i}\hat{\mathbf{o}}^\ell y^\ell$ and replace $\mathbf{o}^{\ell\ell}$ by $\hat{\mathbf{o}}^{\ell\ell}$ in the type of *whom* and of the passive auxiliary *be*, as illustrated by the following examples:

(9.1)
$$\begin{array}{c} \textit{he will be told to go} \\ \pi_3(\pi^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{j}\hat{\mathbf{o}}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2\hat{\mathbf{o}}^\ell\bar{\mathbf{j}}^\ell)(\bar{\mathbf{j}}\mathbf{j}^\ell)\mathbf{i} \end{array} \rightarrow \mathbf{s}_1\hat{\mathbf{o}}^{\ell\ell}\hat{\mathbf{o}}^\ell \rightarrow \mathbf{s}_1$$

(9.2)
$$\begin{array}{c} \textit{whom will I tell to go ?} \\ (\overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_1\mathbf{j}^\ell\pi^\ell)\pi_1(\mathbf{i}\hat{\mathbf{o}}^\ell\bar{\mathbf{j}}^\ell)(\bar{\mathbf{j}}\mathbf{j}^\ell)\mathbf{i} \\ \rightarrow\ \overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\mathbf{j}^\ell\mathbf{i}\hat{\mathbf{o}}^\ell\ \rightarrow\ \overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\hat{\mathbf{o}}^\ell \rightarrow \overline{\mathbf{q}} \end{array}$$

We may even regard the new types of *whom* and the passive auxiliary *be* as refinements of the old types if we postulate $\hat{\mathbf{o}} \rightarrow \mathbf{o}$. But this would demand of English speakers the mathematical sophistication to derive $\hat{\mathbf{o}}^{\ell\ell} \rightarrow \mathbf{o}^{\ell\ell}$ from $\hat{\mathbf{o}} \rightarrow \mathbf{o}$.

We summarize the strategy of Section 6 by the following metarule:

IV. When $y = \mathbf{j}, \bar{\mathbf{j}}, \mathbf{r}$ etc, we may assign to verbs of type $\mathbf{i}y^\ell\mathbf{o}^\ell$ the new type $\mathbf{i}\hat{\mathbf{o}}^\ell y^\ell$ and augment the types of *whom* and of the passive auxiliary *be* by allowing $\hat{\mathbf{o}}^{\ell\ell}$ in place of $\mathbf{o}^{\ell\ell}$.

We may attempt to justify Metarule IV by a mathematical argument. Putting $\mathbf{o}_y = y^r\mathbf{o}y$, we infer $\mathbf{o}_y^\ell = y^\ell\mathbf{o}^\ell y^{r\ell} = y^\ell\mathbf{o}^\ell y$, hence

$$y^\ell\mathbf{o}^\ell \rightarrow y^\ell\mathbf{o}^\ell yy^\ell \rightarrow \mathbf{o}_y^\ell y^\ell\ .$$

If we now postulate $\hat{\mathbf{o}} \rightarrow \mathbf{o}_y$, we may infer $\mathbf{o}_y^\ell \rightarrow \hat{\mathbf{o}}^\ell$, hence the verb in question has type $\mathbf{i}y^\ell\mathbf{o}^\ell \rightarrow \mathbf{i}\hat{\mathbf{o}}^\ell y^\ell$, as required. Unfortunately, $\mathbf{o}_y$ is not a basic type, so we ought not adopt this postulate.

10. The invisible dative.

There are words, such as *give* and *show*, which require two objects, one direct and one indirect.[9)]

(10.1)
$$\begin{array}{c} \textit{I will give her a book} \\ \pi_1(\pi^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{i}\mathbf{o}^\ell\mathbf{o}'^\ell)\mathbf{o}'\ (\overline{\mathbf{c}}\mathbf{c}^\ell)\mathbf{c} \end{array} \rightarrow \mathbf{s}_1\mathbf{o}^\ell\overline{\mathbf{c}} \rightarrow \mathbf{s}_1$$

recalling that $\overline{\mathbf{c}} \rightarrow \mathbf{o}$. Here

$$\mathbf{o}' = \text{ indirect object.}$$

Since every word, such as *her*, denoting a direct object can also represent an indirect one, we must postulate $\mathbf{o} \rightarrow \mathbf{o}'$.

Now (10.1) can be rephrased as follows:

(10.2)
$$\begin{array}{c} \textit{I will give a book to her} \\ \pi_1(\pi^r\mathbf{s}_1\mathbf{j}^\ell)([\mathbf{i}\mathbf{o}^\ell)(\overline{\mathbf{c}}\mathbf{c}^\ell)\mathbf{c}(\mathbf{j}^r\mathbf{i}\mathbf{o}^\ell)\mathbf{o} \end{array} \begin{array}{l} \rightarrow\ \mathbf{s}_1\mathbf{j}^\ell[\mathbf{i}\mathbf{j}^r\mathbf{i} \\ \rightarrow \mathbf{s}_1\mathbf{j}^\ell\mathbf{i} \rightarrow \mathbf{s}_1 \end{array}$$

where *give* is now viewed as an ordinary transitive verb, requiring, just one complement, and the prepositional phrase *to her* can be replaced by *for her* or even by *gladly*. The left square bracket serves as a reminder that the sentence does not end after *book*.

Since slavery has been abolished, the direct object of *give* is usually non-human; so, when asking for the direct object, semantics requires that *whom* be replaced by *what*. However, we can say

(10.3)
whom will I show her ?
$$(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_1\ (\mathbf{i}\mathbf{o}^{\ell}\mathbf{o}'^{\ell})\ \mathbf{o}' \ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}\ \mathbf{i}\mathbf{o}^{\ell} \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\ \mathbf{o}^{\ell}\ \to\ \overline{\mathbf{q}}$$

When asking for the indirect object, it seems more appropriate to start from (11.2) and arrive at

(10.4)
whom will I give a book to − ?
$$(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_1([\mathbf{i}\mathbf{o}^{\ell})(\overline{\mathbf{c}}\mathbf{c}^{\ell})\mathbf{c}(\mathbf{j}^{r}\mathbf{j}\mathbf{o}^{\ell}) \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}[\mathbf{i}\ \mathbf{j}^{r}\mathbf{j}\mathbf{o}^{\ell} \to \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}\ \mathbf{j}\mathbf{o}^{\ell} \to \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell} \to \overline{\mathbf{q}}$$

or, more pedantically,

(10.5)
to whom will I give a book ?
$$(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\overline{\mathbf{q}}^{\ell})(\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_1(\mathbf{i}\mathbf{o}^{\ell})(\overline{\mathbf{c}}\mathbf{c}^{\ell})\mathbf{c} \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell\ell}\mathbf{o}^{\ell}\mathbf{j}^{\ell}\ \mathbf{i} \to \overline{\mathbf{q}}$$

Can we apply Metarule IV to *give* of type $\mathbf{i}\mathbf{o}^{\ell}\mathbf{o}'^{\ell}$? First, we would have to point out that $\mathbf{o}'^{\ell} \to \mathbf{o}^{\ell}$ since $\mathbf{o} \to \mathbf{o}'$, reluctantly recognizing contravariance of adjunction.[10] Thus, *give* also has type $\mathbf{i}y^{\ell}\mathbf{o}^{\ell}$, where $y = \mathbf{o}$. Metarule IV would then assert that *give* also has type $\mathbf{i}\hat{\mathbf{o}}^{\ell}\mathbf{o}^{\ell}$ and predict

(10.6)
she will be given a book
$$\pi_3\ (\pi^{r}\mathbf{s}_1\mathbf{j}^{\ell})(\mathbf{j}\hat{\mathbf{o}}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\hat{\mathbf{o}}^{\ell}\mathbf{o}^{\ell})(\overline{\mathbf{c}}\mathbf{c}^{\ell})\mathbf{c} \quad\to\ \mathbf{s}_1\hat{\mathbf{o}}^{\ell\ell}\hat{\mathbf{o}}^{\ell}\ \to\ \mathbf{s}_1$$

thus explaining why the indirect object can become the subject in the passive in English.

Metarule IV would also predict

(10.7)
* *whom will I give a book ?*
$$(\overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_1(\mathbf{i}\hat{\mathbf{o}}^{\ell}\mathbf{o}^{\ell})(\overline{\mathbf{c}}\mathbf{c}^{\ell})\mathbf{c} \quad\to\ \overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\mathbf{j}^{\ell}\mathbf{i}\hat{\mathbf{o}}^{\ell} \to\ \overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\hat{\mathbf{o}}^{\ell}\ \to\ \overline{\mathbf{q}}$$

However, I am told that this is incorrect. Why is this so? One conceivable explanation is this:

if we replace *give* by *show* and *a book* by *her*, we will obtain *whom will I show her*? but this time asking for the indirect object, in conflict with (10.3), which was asking for the direct object.[10]

11. Silent *whom.*

When the object relative pronoun *whom/which* introduces a restrictive relative clause modifying a noun, it may be left out altogether, as in

(11.1)
girls ∅ *(I have known −)*
$$\mathbf{p}\ (\mathbf{p}^{r}\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\ \pi_1\ (\pi_1^{r}\mathbf{s}_1\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell}) \quad\to\ \mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\ \mathbf{s}_1\mathbf{o}^{\ell} \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell}\ \to\ \mathbf{p}$$

Rather than assigning the type $\mathbf{p}^r\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ to the empty string, I have proposed elsewhere [2001] that we should attach an invisible ending of type $\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ to *girls* of type $\mathbf{p}$, thus yielding the type $\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ as an alternative type for *girls*.

There is a problem with this proposal when the noun is modified by a prepositional phrase, as in

girls with glasses I have known .

We do not wish to attach the invisible ending to *glasses* in this context, since the relative clause is intended to modify *girls* and not *glasses*. One solution, perhaps an overly artificial one, is to modify the preposition *with*, by saying that prepositions of type $x^r x\mathbf{o}^\ell$, with $x = \mathbf{p}, \mathbf{c}$ or $\mathbf{m}$, may also have an alternative type $x^r x\mathbf{o}^{\ell\ell}\mathbf{s}^\ell\mathbf{o}^\ell$, as in

(11.2)
$$\begin{array}{ll} \textit{girls with glasses} & (\textit{I have known} -) \\ \underline{\mathbf{p}\ (\mathbf{p}^r}\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell\underline{\mathbf{o}^\ell)\ \mathbf{o}}\ (\mathbf{s}_1\mathbf{o}^\ell) & \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\underline{\mathbf{s}^\ell\ \mathbf{s}_1}\mathbf{o}^\ell \\ & \to\ \mathbf{p}\underline{\mathbf{o}^{\ell\ell}\ \mathbf{o}^\ell}\ \to\ \mathbf{p} \end{array}$$

A similar problem arises with co-ordinate conjunctions such as *and*, as in

(11.3)
$$\begin{array}{c} \textit{boys and girls } (\textit{I have known} -) \\ \underline{\mathbf{p}\ (\mathbf{p}^r}\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell\underline{\mathbf{p}^\ell)\ \mathbf{p}}\ (\mathbf{s}_1\mathbf{o}^\ell)\ \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\underline{\mathbf{s}^\ell\ \mathbf{s}_1}\mathbf{o}^\ell\ \to\ \mathbf{p}\underline{\mathbf{o}^{\ell\ell}\mathbf{o}^\ell}\ \to\ \mathbf{p} \end{array}$$

where it is the type of the conjunction *and* that has to be modified. Here is a tentative metarule:

V. For nouns of type $x = \mathbf{c}, \mathbf{p}$ or m, prepositions of type $x^r x\mathbf{o}^\ell$ and conjunctions of type $x^r x x^\ell$, the single or middle x may be replaced by $x\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$.

Now what about the following?

girls that I have known .

If *that* is just a variant of *whom*, like *which*, there is no problem. However, McCawley [1988] suggests that, in this context, *that* is not a relative pronoun, but a complementizer. If so, we should amend Metarule V by adding

V′. .....or x may be replaced by $x\mathbf{o}^{\ell\ell}\overline{\mathbf{s}}^\ell$.

Thus we analyze

(11.4)
$$\begin{array}{c} \textit{girls that I have known} - \\ (\mathbf{p}\mathbf{o}^{\ell\ell}\underline{\overline{\mathbf{s}}^\ell)\ (\overline{\mathbf{s}}}\mathbf{s}^\ell)\underline{\pi_1\ (\pi_1^r}\mathbf{s}_1\underline{\mathbf{p}_2^\ell)(\mathbf{p}_2}\mathbf{o}^\ell) \\ \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\underline{\mathbf{s}^\ell\ \mathbf{s}_1}\mathbf{o}^\ell\ \to \mathbf{p}\underline{\mathbf{o}^{\ell\ell}\mathbf{o}^\ell}\ \to\ \mathbf{p} \end{array}$$

In support of McCawley's proposal, let me point out that in

girls with whom he went ,

whom cannot be replaced by *that*.[11)]

The reader may wonder why we have not adopted an easier way of accounting for the missing object relative pronoun by postulating $x \to x\mathbf{o}^{\ell\ell}\mathbf{s}^\ell$ or, equivalently, $x\mathbf{s}\mathbf{o}^\ell \to x$. Admitting such postulates would imply that the pregroup under consideration is no longer free, and we would lose the decision procedure implicit in the Switching Lemma.

Replacing such a postulate by a metarule allows us to retain the decision procedure, at least formally. It is, however, well-known that ellipsis of relative pronouns rapidly leads to sentences which are hard to parse. Consider, for example, the noun phrase

(11.5)
$$\begin{array}{l} \textit{police police police} \\ (\mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\ \mathbf{p}\ (\pi_2^r\mathbf{s}_1\mathbf{o}^{\ell}) \ \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\mathbf{s}_1\mathbf{o}^{\ell} \\ \qquad\qquad\qquad\qquad\quad \to\ \mathbf{p}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell}\ \to\ \mathbf{p} \end{array}$$

meaning

police [whom] police control .

The reader interested in mathematical puzzles may wish to prove, with the help of (11.5), that, for any positive integer n, $\textit{police}^{2n+1}$ may be parsed grammatically as a declarative sentence in $n!$ distinct ways.[12)].

12. Long distance dependency.

One remarkable thing about the types of *whom* is that they can act at a distance:

(12.1)
$$\begin{array}{c} \textit{you know the girl whom I said I saw} - \\ \pi_2(\pi_2^r\mathbf{s}_1\mathbf{o}^{\ell})(\overline{\mathbf{c}}\mathbf{c}^{\ell})[\mathbf{c}(\mathbf{c}^r\mathbf{c}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell})\pi_1(\pi^r\mathbf{s}_2\mathbf{s}^{\ell})\pi_1(\pi^r\mathbf{s}_2\mathbf{o}^{\ell}) \\ \to\ \mathbf{s}_1\mathbf{c}^{\ell}\ \mathbf{c}\mathbf{o}^{\ell\ell}\mathbf{s}^{\ell}\mathbf{s}_2\mathbf{s}^{\ell}\mathbf{s}_2\mathbf{o}^{\ell}\ \to\ \mathbf{s}_1\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell}\to\mathbf{s}_1 \end{array}$$

(12.2)
$$\begin{array}{c} \textit{whom do you know whether I gave the book to} - ? \\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{i}^{\ell}\pi^{\ell})\pi_2(\mathbf{i}\mathbf{r}^{\ell})(\mathbf{r}\mathbf{s}^{\ell})\pi_1(\pi^{\ell}\mathbf{s}_2\mathbf{j}^{\ell}[\mathbf{j}\mathbf{o}^{\ell})(\overline{\mathbf{c}}\mathbf{c}^{\ell})\mathbf{c}(\mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell}) \\ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{i}^{\ell}\ \mathbf{i}\mathbf{s}^{\ell}\mathbf{s}_2\mathbf{j}^{\ell}[\mathbf{j}\ \mathbf{j}^r\mathbf{i}\mathbf{o}^{\ell} \\ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}\ \mathbf{i}\mathbf{o}^{\ell}\ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell}\ \to\ \overline{\mathbf{q}} \end{array}$$

Unfortunately, our method wrongly predicts the acceptability of

(12.3)
$$\begin{array}{c} {}^{*}\textit{whom will you see him and} - ? \\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^{\ell})(\mathbf{q}_1\mathbf{j}^{\ell}\pi^{\ell})\pi_2(\mathbf{i}[\mathbf{o}^{\ell})[\mathbf{o}\ (\mathbf{o}^r\mathbf{o}\mathbf{o}^{\ell}) \\ \to\ \overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{j}^{\ell}\ \mathbf{i}\mathbf{o}^{\ell}\ \mathbf{o}\mathbf{o}^{\ell}\to\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell}\to\overline{\mathbf{q}} \end{array}$$

The same problem arises in mainstream grammars and is then handled by "restrictions on transformations" or, more recently, "obstacles to movement". Anne Preller [t.a.] has shown that (12.3) will actually turn out to be ungrammatical if a clever type assignment, different from the present one, is adopted. My own contention has been that the unacceptability of (12.3) should be accounted for by processing difficulties. As soon as the hearer has analyzed the first six words of (12.3), her type calculation has arrived at $\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}[\mathbf{o}^{\ell}[\mathbf{o}\mathbf{o}^r\mathbf{o}^{\ell}$. To reduce this to $\overline{\mathbf{q}}$, she must forgo two consecutive contractions, $\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell}\to 1$ and $\mathbf{o}^{\ell}\mathbf{o}\to 1$. I have argued in [2001] that this might prove too difficult for the hearer. In a later paper [2004], I proposed two other algebraically formulated constraints that may replace the traditional ones.

However, the other two constraints can be reduced to the above *block constraint*, which asserts that $[x[y$ is too hard to process, when x and y are simple types. A second constraint had asserted that one cannot process two consecutive double adjoints. Now, if we believe with Sergeant Lewis that there are no triple adjoints in English, we may write a double adjoint as $[x^{\ell\ell}$, x being a basic type, to remind us that it cannot be annihilated from the left. Thus, two consecutive double adjoints take the form $[x^{\ell\ell}[y^{\ell\ell}$, hence are already covered by the block constraint. Finally, assigning the type $y^r y y^\ell$ to the coordinating conjunction *and*, I had proposed a constraint ruling out $x^{\ell\ell}y^\ell[yy^r yy^\ell$. I now suggest changing the type of the conjunction to $y^r y[y^\ell$, then

$$[x^{\ell\ell}y^\ell[\underbrace{yy^r}y[y^\ell \to [x^{\ell\ell}\underbrace{y^\ell y}[y^\ell \to [x^{\ell\ell}[y^\ell$$

will also be ruled out by the block constraint.

13. Concluding remarks.

The pregroup grammars I now advocate allow us to analyze a sentence linearly, proceeding step by step from left to right. This is in contrast to the two-dimensional proofs of earlier categorial grammars and their multimodal modifications, as well as to the page-filling trees of generative transformational grammars. In my opinion, pregroup grammars are particularly suitable for computations, which model the kind of calculations that go on, albeit subconsciously, in the brains of humans engaged in discourse.

In this article, we have concentrated on a tiny fragment of English grammar, focussing on the object pronoun *who(m)*. In other articles, I have approached different aspects of English grammar with the help of pregroups, although the particular types I assigned to English words may have changed over time. Similar work has been carried out on some other languages: Arabic, French, German, Italian, Japanese, Latin, Polish and Turkish. Double adjoints have proved useful so far in modern European languages only, accounting for Chomskyan traces and Romance clitic pronouns. Triple adjoints appear for the first time in the present article.

The pregroups we employ are freely generated by a poset of *basic* types, which may vary from one language to another and incorporate features such as tense, person, number and case. To each word of the language we assign one or more *types*, namely strings of *simple types*, which are formed from basic types by taking repeated adjoints. English words tend to have many types. For example, the word *sound* can be a noun, an adjective or a verb, and as a verb it can be an infinitive or the first or second person of the present tense. The object pronoun *who(m)* can have many types, depending on its function. Of these, we have considered here

$$\bar{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell,\ \mathbf{r}\mathbf{o}^{\ell\ell}\mathbf{s}^\ell,\ \mathbf{r}\mathbf{o}^{\ell\ell}\bar{\mathbf{j}}^\ell,\ x^r x\mathbf{o}^{\ell\ell}\mathbf{s}^\ell,$$

where $x = \mathbf{c}, \mathbf{p}, \bar{\mathbf{c}}, \bar{\mathbf{p}}$, as well as some others, which can be derived from these systematically by certain metarules. These allow us to replace $\mathbf{o}^{\ell\ell}$ by $\hat{\mathbf{o}}^{\ell\ell}$.

Why the metarules? Roughly speaking, they serve a purpose similar to that of Chomsky's transformations, with one significant difference: transformations apply to labelled bracketed strings or to trees, whereas our metarules apply to words in the dictionary.

Only the first few steps of a hopefully promising program have been carried out so far, and many problems of interest to contemporary linguists have not yet been addressed. Controversial reactions are expected from adherents of other methodologies. There has been an ongoing discussion with adherents of the multi-model type logical approach. So far, there has not yet been any reaction from other schools, such as Gazdar's generalized phrase structure grammars and Chomsky's minimalist program, unless such reactions were hidden in anonymous referees' reports.

Endnotes

1) Actually, the first [1958] paper investigated a residuated semigroup.

2) This we learned from Avarind Joshi.

3) Actually, in the syntactic calculus, this was originally analyzed as

$$(S/(N\backslash S))(N\backslash S/N)((S/N)\backslash S)$$

where only two basic types, S for sentence and N for noun phrase, were used.

4) In other European languages, we would require also types π_4, π_5 and π_6 for the three persons of the plural.

5) Some friendly critics have urged me to drop the antisymmetry law, thus defining a pregroup as a quasi-ordered monoid with adjoints. Then the word "unique" in (3.1) would have to be replaced by "unique up to $\leftrightarrow$". Following the "categorical imperative", one should replace pregroups by "compact monoidal categories". Then one could even say "unique up to isomorphism", although $\leftrightarrow$ does not always give rise to an isomorphism.

6) With the help of the Switching Lemma, Buszkowski [2002] has shown that the logical system corresponding to free pregroups, namely compact bilinear logic, enjoys the cut-elimination property. He also proved [2001] that grammars based on free pregroups are context-free.

7) The transitive verb *have* does have them.

8) The copula *be* does.

9) Such words tend to denote causation. Thus *give* means "let have" and *show* means "let see".

10) Or else one could take $\mathbf{o}' = \mathbf{o}$ in the first place. In contrast to English, the analogue of (10.7) is acceptable in German, the analogue of (10.3) is not.

11) Whereas English spelling disguises the distinction between the complementizer *that* and the relative pronoun *that*, German distinguishes between *dass* and *das*.

12) A similar parsing in case $n = 2$ is implicit in an example given by Pinker (page 210).

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