Exploring feature agreement in French with parallel pregroup computations.

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Abstract

One way of coping with agreement of features in French is to perform two parallel computations, one in the free pregroup of *syntactic* types, the other in that of *feature* types. Technically speaking, this amounts to working in the direct product of two free pregroups.

1. Introduction.

Agreement of features is a requirement in many languages, but may differ from one language to another. Thus, Latin requires an adjective to agree with the noun it modifies in gender, number and case, whether it occurs in attributive or in predicative position. In French, case has disappeared, except in pronouns. German retains case, but insists on agreement only for attributes and not for predicates. English retains only few features and exhibits no agreement between adjectives and nouns.

In [BL2001], we made a first attempt to describe French sentence structure with the help of a pregroup grammar, but we deliberately ignored feature agreement. Since then, several people have addressed this problem and attempted to fill the gap. Anne Preller and Violaine Prince [2008] have successfully adopted a strategy of adding subscripts denoting gender and number to many, if not most types of words in a sentence, even when the words do not express these features explicitly.[1]

On the other hand, students of Ed Stabler and Brendan Gillon[2] have adopted a different strategy, carrying out calculations on features in parallel with those on the original syntactic types, working essentially in the direct product of two or more free pregroups, as pointed out by Telyn Kusalik [2008]. It is this strategy that will be adopted ultimately in the present article, even if not in exactly the same manner. The present approach uses two free pregroups only and retains the syntactic types adopted previously [loc.cit.], with one small amendment to the types of reflexive pronouns. It has the advantage of strictly extending the generative power of pregroup grammars, which has been proved by Buszkowski to be equivalent to that of context-free ones. By working with two free pregroups in parallel, one is dealing with the intersection of two context-free languages, and this includes examples of languages that have been proved to be non-context-free.

2. Recalling pregroup grammars.

A *pregroup* is a partially ordered monoid (semigroup with unity element) endowed with two unary operations, called *left* and *right adjoint* respectively, satisfying the *contraction* rules

$$a^{\ell}a \rightarrow 1, \ aa^r \rightarrow 1$$

and the *expansion* rules
$$1 \to aa^\ell, \ 1 \to a^r a,$$
the partial order being denoted by an arrow.

One can show that adjoints are unique and that adjunction is contravariant:

$$\text{if } a \to b \text{ then } b^\ell \to a^\ell \text{ and } b^r \to a^r.$$

Moreover, once can prove that

$$1^\ell = 1, \ (ab)^\ell = b^\ell a^\ell, \ a^{r\ell} = a$$

and similarly with ℓ replaced by r and conversely.

A *pregroup grammar* of a language, as hitherto conceived, assigns to each word or morpheme of the language (as well as to certain "inflectors", to be discussed in the next section) one or more (compound) types. These are elements of the pregroup *freely generated* from a partially ordered set of *basic* types. With any basic type a we associate the *simple* types

$$\ldots \ a^{\ell\ell}, \ a^\ell, \ a, \ a^r, \ a^{rr}, \ \ldots$$

By a compound type, or just *type*, we mean a string of simple types, including the empty string 1. As it turns out [L1992], as long as one is interested only in showing that a string of types reduces to a simple type, only contractions are needed. However, expansions do play a rôle in occasional theoretical arguments.

3. Review of previous work on French grammar.

The French verb V has $7 \times 6 = 42$ finite forms V_{jk}, where $j = 1$ to 7 denotes the seven simple tenses and $k = 1$ to 6 the three persons singular followed by the three persons plural. It had been shown in [L1976] how these 42 forms can be calculated from certain given "radicals". It is convenient to write
$$C_{jk} V \to V_{jk},$$
where C_{jk} is what has been called an "inflector", V is a verb described by its infinitive and the arrow is read as "rewrite".[3] For example,

$$C_{14} \ dormir \ \to \ dormons,$$
$$C_{23} \ pouvoir \ \to \ pouvait.$$

A crucial strategy in [BL2001] had been to apply inflectors not only to bare infinitives, but also to *extended infinitives*, incorporating preverbal oblique pronouns, as in

$$C_{15} \ la + lui + donner \ \to \ la \ lui \ donnons.$$

Here C_{15} replaces a visible modal verb form (*devons, pouvons* or *voulons*) and may itself be regarded as an invisible inflected modal verb. For easier reading, we will largely confine attention to visible modal verbs in what follows.

The use of double adjoints, such as $a^{\ell\ell}$, is crucial for our type assignment to preverbal oblique pronouns.[4] To illustrate our approach, consider the following example, where each French word has been assigned a compound type written below it:

(3.1)
$$nous\ pouvons\ la\ lui\ donner$$
$$\pi_4\ (\pi_4^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}'\omega^{\ell\ell}\mathbf{i}^\ell)(\mathbf{i}\omega^\ell\mathbf{o}^\ell) \quad \rightarrow\ \mathbf{s}_1$$

We have used the following basic types, ranging over diverse grammatical categories borrowed from traditional syntax and morphology.

π_k for subject pronoun (in nominative case), where $k = 1$ to 6 stands for the three persons singular followed by the three persons plural;

$\mathbf{s}_j$ for declarative sentence in the j-th tense, where $j = 1$ stands for the present, $j = 2$ for the imperfect past, etc;

$\mathbf{o}$ for direct object $= COD$ (in accusative case);
ω for indirect object $= COI$ (in dative case);
$\mathbf{i} \underset{\neq}{\rightarrow} \mathbf{i}' \underset{\neq}{\rightarrow} \mathbf{j}$ for infinitives or infinitival phrases.

The underlinks in (3.1) going back to [Harris 1968]) indicate contractions, e.g.

$$\pi_4 \pi_4^r \rightarrow 1,$$
$$\mathbf{j}^\ell \mathbf{i}' \rightarrow \mathbf{j}^\ell \mathbf{j} \rightarrow 1,\ \text{etc.}$$

The three distinct types of infinitives were needed to ensure the correct order of preverbal oblique pronouns, e.g. to avoid

(3.2)
$$^*nous\ pouvons\ lui\ la\ donner,$$
$$(\mathbf{i}\omega^{\ell\ell}\mathbf{i}^\ell)(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}^\ell)$$

since $\mathbf{i}^\ell\mathbf{i}' \nrightarrow 1$. Indeed, $\mathbf{i}^\ell\mathbf{i}' \rightarrow 1$ would imply

$$\mathbf{i}' = 1\mathbf{i}' \rightarrow \mathbf{i}\mathbf{i}^\ell\mathbf{i}' \rightarrow \mathbf{i}1 = \mathbf{i},$$

making use of the expansion $1 \rightarrow \mathbf{i}\mathbf{i}'$. But this would imply $\mathbf{i}' = \mathbf{i}$, contrary to our stipulation.

In order to explain

$$nous\ la\ lui\ donnons, \quad ^*nous\ lui\ la\ donnons$$

we proceed as above, but replace the modal *pouvons* by the naked inflector C_{14} of type $\pi_4^r \mathbf{s}_1 \mathbf{j}^\ell$. In general, the inflector $C_{\mathbf{j}k}$ has type $\pi_k^r \mathbf{s}_j \mathbf{j}^\ell$.

Our type assignment also admits

(3.3)
$$il\ peut\ me\ la\ donner$$
$$\pi_3(\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\omega^{\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}\mathbf{o}^\ell\omega^\ell)$$

and rejects

$$^*il\ peut\ la\ me\ donner.$$
$$(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell)(\mathbf{j}\mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell)$$

3

We recall the following type assignments from [BL2001]:

le, la, les: $\mathbf{i'o}^{\ell\ell}\mathbf{i'}^{\ell}$,

lui, leur: $\mathbf{i'}\omega^{\ell\ell}\mathbf{i}^{\ell}$,

me, te, se, nous, vous: $\quad\mathbf{jo}^{\ell\ell}\mathbf{i}^{\ell}$ in accusative case,
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\mathbf{j}\omega^{\ell\ell}\mathbf{i'}^{\ell}$ in dative case.

The typing of the reflexive preverbal oblique pronouns will be slightly amended in Sections 6 to 8 below.

We also make use of the basic type $\mathbf{p}_2$ for past participles of intransitive verbs requiring *avoir* for the compound past (= perfect tense), $\mathbf{p}'_2$ for past participles requiring *être* instead. For example, assuming the person addressed is female,

(3.4)
$$\begin{array}{c} \textit{tu peux avoir dormi } (^*\textit{dormie}) \\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{i}^{\ell})(\mathbf{ip}_2^{\ell})\mathbf{p}_2 \;\rightarrow\; \mathbf{s}_1 \end{array}$$

(3.5)
$$\begin{array}{c} \textit{tu peux être venue } (^*\textit{venu}) \\ \pi_1(\pi_1^r\mathbf{s}_1\mathbf{j}^{\ell})(\mathbf{ip}_2'^{\ell})\mathbf{p}'_2 \;\rightarrow\; \mathbf{s}_1 \end{array}$$

or the more intricate

(3.6)
$$\begin{array}{c} \textit{nous pouvons la lui avoir donnée } (^*\textit{donné}) \\ \pi_4(\pi_4\mathbf{s}_1\mathbf{j}^{\ell})(\mathbf{i'o}^{\ell\ell}\mathbf{i'}^{\ell})(\mathbf{i'}\omega^{\ell\ell}\mathbf{i}^{\ell})(\mathbf{ip}_2^{\ell})(\mathbf{p}_2\omega^{\ell}\mathbf{o}^{\ell}) \;\rightarrow\; \mathbf{s}_1 \end{array}$$

The choice of gender in (3.4) to (3.6) still requires an explanation in terms of pregroups.

Note that the auxiliary verb *être* serves several additional functions, as a copula of type $\mathbf{ia}^{\ell}$, with

$\mathbf{a}$ for predicative adjective,

and as a passive auxiliary of type $\mathbf{io}^{\ell\ell}\mathbf{p}_2^{\ell}$.

(3.7)
$$\begin{array}{cc} \begin{array}{c} \textit{nous voulons être heureux,} \\ \pi_4\;(\pi_4^r\mathbf{s}_1\mathbf{j}^{\ell})\;(\mathbf{ia}^{\ell})\;\mathbf{a} \end{array} & \rightarrow\; \mathbf{s}_1 \end{array}$$

(3.8)
$$\begin{array}{cc} \begin{array}{c} \textit{elle voulait être embracée} \\ \pi_3(\pi_3^r\mathbf{s}_2\mathbf{j}^{\ell})(\mathbf{io}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell}) \end{array} & \rightarrow\; \mathbf{s}_2 \end{array}$$

(3.9)
$$\begin{array}{cc} \begin{array}{c} \textit{elles pouvaient avoir été mangées} \\ \pi_6\;(\pi_6^r\mathbf{s}_2\mathbf{j}^{\ell})(\mathbf{ip}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{o}^{\ell}) \end{array} & \rightarrow\; \mathbf{s}_2 \end{array}$$

referring to *les pommes*. Again, the genders and numbers in (3.7) to (3.9) ask for an explanation, which will be given in Sections 4 and 5 below.

4. Feature types of adjectives.

French has retained the features gender and number for both attributive and predicative adjectives.[6)] Rules for displaying the features masculine singular, feminine singular, masculine plural and feminine plural may be found in the Bescherelle, showing how to form for example:

$$\textit{ancien, ancienne, anciens, anciennes,}$$
$$\textit{heureux, heureuse, heureux, heureuses,}$$
$$\textit{triste, triste, tristes, tristes,}$$
$$\textit{beau/bel, belle, beaux, belles.}$$

In [BL2001], the features gender and number were completely ignored, and no attempt was made to eliminate e.g.

(4.1)
$$\textit{elle veut être *heureux.}$$
$$\pi_3 \ (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{ia}^\ell) \ \mathbf{a} \qquad \rightarrow \ \mathbf{s}_1$$

Matters might be improved by adding a subscript f to indicate femininity:

(4.2)
$$\textit{elle veut être heureuse}$$
$$\pi_{3f} \ (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{ia}^\ell) \ \mathbf{a}_f \qquad \rightarrow \ \mathbf{s}_1$$

provided we stipulate

$$\pi_{3f} \rightarrow \pi_3, \ \mathbf{a}_f \rightarrow \mathbf{a}$$

so that e.g.

$$\mathbf{a}^\ell \mathbf{a}_f \rightarrow \mathbf{a}^\ell \mathbf{a} \rightarrow 1.$$

Unfortunately, this would not explain how a speaker, about to utter the last word, would still remember that the subject was feminine and refrain from saying *heureux* instead.

In the meantime, this problem has been addressed in different ways. For example, Anne Preller and Violaine Prince [2008] have adopted a strategy of attaching the subscript f to the types of intermediate words, even though these may be gender neutral. Their method may be illustrated by

(4.3)
$$\textit{elle veut être heureuse,}$$
$$\pi_{3f}(\pi_{3f}^r \mathbf{s}_1 \mathbf{j}_f^\ell)(\mathbf{i}_f \mathbf{a}_f^r)\mathbf{a}_f \rightarrow \ \mathbf{s}_1$$

(without necessarily following their type assignment, which differs significantly from [BL2001]).

Another strategy has been proposed by Ed Stabler and Brendan Gillon (and was adopted by some of their students), namely to carry out a parallel computation on features, thus working in two (or more) free pregroups at the same time. We will distinguish the new *feature types* from the old *syntactic types* and write the former below the latter:

(4.4)

	elle	*peut*	*être*	*heureuse*	
	π_3	$(\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell)$	$(\mathbf{ia}^\ell)$	$\mathbf{a}$	$\rightarrow \ \mathbf{s}_1$
	π_{3f}			π_{3f}^r	$\rightarrow \ 1$

Here the following type assignments have been adopted:

elle: $\begin{bmatrix} \pi_3 \\ \pi_{3f} \end{bmatrix}$, where the subscript $3f$ serves to indicate that *elle* is third person feminine;

peut: $\begin{bmatrix} \pi_3^r \mathbf{s}_1 \mathbf{j}^\ell \\ 1 \end{bmatrix}$;

$\hat{e}tre$: $\begin{bmatrix} \mathbf{ia}^{\ell} \\ 1 \end{bmatrix}$;

$heureuse$: $\begin{bmatrix} \mathbf{a} \\ \pi^r_{3f} \end{bmatrix}$, where the subscript $3f$ indicates that $heureuse$ is singular and feminine.

We have introduced a second pregroup freely generated from basic feature types π_{kg}, π_{ng} (and others to be met presently) where $k = 1$ to 6 stands for person,

$n = s$ or p stands for number,

$s =$ singular and $p =$ plural, and

$g = m$ or f stands for gender. These are subject to the following postulates:

$$\pi_{kg} \underset{\neq}{\to} \pi_{sg} \text{ if } k = 1 \text{ to } 3,$$

$$\pi_{kg} \underset{\neq}{\to} \pi_{pg} \text{ if } k = 4 \text{ to } 6.$$

In the above example, (4.4), the symbol 1 (denoting the empty string) and the square brackets have been omitted for easier reading on the left side. On the right side the symbol 1 has been retained as a kind of check mark to indicate that the features do agree.

Altogether, we will make use of the basic feature types

$$\pi_{kg}, \ \pi_k, \ \pi_{ng}, \ \pi; \ \pi'_{kg}, \ \pi'_k, \ \pi'_{ng}, \ \pi'$$

and stipulate

$$\begin{array}{ccc} \pi_{kg} & \xrightarrow{\neq} & \pi_k \\ {\scriptstyle\neq}\big\downarrow & & \big\downarrow{\scriptstyle\neq} \\ \pi_{ng} & \xrightarrow{\neq} & \pi \end{array}$$

and similarly with primes added. The irreversible horizontal arrows here forget gender, while the vertical arrows forget person.

We assume that $\pi_x \to \pi'_x$, but $\pi'_x \not\to \pi_y$, where x and y may be kg, k, ng or blank.

Different authors have used different numbers of free pregroups, thus

Preller and Prince use 1,

Kusalik and Vander Klok use 3,

Pedersen and Fowles use 4,

Kobele uses 5 (for Italian).

5. Feature types of past participles.

The past participle in French enters two constructions: forming the compound past (aka perfect tense) and forming the passive of a transitive verb. The former is done with the help of the auxiliary verb $\hat{e}tre$, the latter usually with the help of the auxiliary verb $avoir$, but for certain exceptional intransitive verbs with $\hat{e}tre$. We assign the type $\mathbf{p}_2$ to past participle phrases (including any object complements) in all but the exceptional cases, where the type $\mathbf{p}'_2$ has been assigned in [BL2001]. In what follows we confine attention to $\mathbf{p}_2$.

Here are some examples:

$$
\begin{array}{cccccc}
& \textit{elle} & \textit{veut} & \textit{être} & \textit{embrassée} & (\textit{*embrassé}) \\
(5.1) & \pi_3 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{io}^{\ell\ell} \mathbf{p}_2^\ell) & (\mathbf{p}_2 \mathbf{o}^\ell) & \rightarrow \mathbf{s}_1 \\
& \pi_{3f} & & & \pi_{sf}'^{r} & \rightarrow 1
\end{array}
$$

where *embrassée* has been assigned the feature type π_{sf}^r to ensure that

$$
\pi_{3f} \pi_{sf}'^{r} \rightarrow \pi_{sf} \pi_{sf}'^{r} \rightarrow \pi_{sf}' \pi_{sf}'^{r} \rightarrow 1.
$$

The reader may ignore the prime (alias bar) for the moment.

$$
\begin{array}{ccccccc}
& \textit{nous} & \textit{pouvons} & \textit{avoir} & \textit{embrassé} & \textit{la fille} & (\textit{*embrassée}) \\
(5.2) & \pi_4 & (\pi_4^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{ip}_2^\ell) & (\mathbf{p}_2 \mathbf{o}^\ell) & \mathbf{o} & \rightarrow \mathbf{s}_1 \\
& \pi_{4m} & & (\pi^r \pi_{sm}) & \pi_{sm}'^{r} & & \rightarrow 1
\end{array}
$$

where the past participle phrase *embrassé la fille* has the feature type $\pi_{sm}'^{r}$. The simple type π^r under *avoir* indicates that the gender and number of the subject are now irrelevant and the basic type π_{sm} ensures that the past participle after *avoir* is uninflected, only bearing the feature singular + masculine by default.

$$
\begin{array}{ccccccc}
& \textit{elle} & \textit{doit} & \textit{avoir} & \textit{été} & \textit{embrassée} & (\textit{*embrassé}) \\
(5.3) & \pi_3 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{ip}_2^\ell) & (\mathbf{p}_2 \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell) & (\mathbf{p}_2 \mathbf{o}^\ell) & \rightarrow \mathbf{s}_1 \\
& \pi_{3f} & & (\pi^r \pi_{sm}) & (\pi_{sm}^r \pi^{rr}) & \pi_{sf}'^{r} & \rightarrow 1
\end{array}
$$

where *été* has feature type $\pi_{sm}^r \pi^{rr}$, with a rare occurrence of a double right adjoint. Note that $\pi_{sm}^r \pi^{rr} \nrightarrow 1$, since otherwise
$$
1 = 1^\ell \rightarrow (\pi_{sm}^r \pi^{rr})^\ell = \pi^r \pi_{sm},
$$

which would imply that
$$
\pi = \pi 1 \rightarrow \pi \pi^r \pi_{sm} \rightarrow 1 \pi_{sm} = \pi_{sm},
$$

contrary to the stipulation that $\pi_{sm} \neq \pi$.

6. Third person accusative preverbal pronouns.
The non-reflexive *le, la* and *les* will be assigned the feature types

$$
\pi'^{r} \pi_{sm}', \ \pi'^{r} \pi_{sf}', \ \pi'^{r} \pi_{pg}'
$$

respectively, where $g = m$ or f is assumed to be in back of the speaker's mind. For example,

$$
\begin{array}{ccccc}
& \textit{il} & \textit{veut} & \textit{la} & \textit{embrasser} \\
(6.1) & \pi_3 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{i}' \mathbf{o}^{\ell\ell} \mathbf{i}'^{\ell}) & (\mathbf{io}^\ell) & \rightarrow \mathbf{s}_1 \\
& \pi_{3m} & & (\pi'^{r} \pi_{sf}') & \pi'^{r} & \rightarrow 1
\end{array}
$$

since

$$
\pi_{3m} \pi'^{r} \rightarrow \pi \pi'^{r} \rightarrow \pi' \pi'^{r} \rightarrow 1,
$$

provided we assign the feature type π'^r to the infinitive *embrasser*. However, we cannot justify

$$
\text{(6.2)} \qquad
\begin{array}{ccccc}
il & veut & la & avoir & {}^*embrass\acute{e} \\
\pi_1 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{i'o}^{\ell\ell}\mathbf{i'}^\ell)(\mathbf{ip}_2^\ell) & & (\mathbf{p}_2\mathbf{o}^\ell) \\
\pi_{3m} & & (\pi'^r \pi'_{sf}) \, (\pi^r \pi_{sm}) & & \pi'^r_{sm}
\end{array}
$$

since $\pi'_{sf}\pi^r \to 1$ would imply that

$$
\pi'_{sf} = \pi'_{sf}1 \to \pi'_{sf}\pi^r\pi \to 1\pi = \pi,
$$

contrary to the convention near the end of Section 4. The correct version

$$
\text{(6.3)} \qquad
\begin{array}{cccc}
& il \ veut \ la \ avoir \ embrass\acute{e}e & \\
\pi_{3m} & (\pi^r\pi'_{sf}\pi^\ell\pi) \ (\pi^r\pi_{sm})\pi'^r_{sf} & \to 1
\end{array}
$$

requires a new feature type for *la*, namely

    la: $\pi^r\pi'_{sf}\pi^r\pi$

and similarly for *le* and *les*, with sf replaced by sm and pg respectively.

We note that

$$
\begin{array}{ccc}
{}^*\pi'^r\pi'_{ng}\pi^\ell\pi & \rightharpoondown & \pi'^r\pi_{ng} \\
\big\downarrow & & \big\downarrow \\
\pi^r\pi'_{ng}\pi^\ell\pi & \longrightarrow & \pi^r\pi'_{ng}
\end{array}
$$

since $\pi \to \pi'$ implies $\pi'^r \to \pi^r$. The common type ${}^*\pi'^r\pi'_{ng}\pi^\ell\pi$ is now obsolete (if it ever existed).

The reflexive pronoun *se* may be accusative or dative. In the former case it will have type

$$
\left[\begin{array}{c}
\mathbf{jo}^{\ell\ell}(\mathbf{o}^{\ell\ell\ell})\mathbf{i}^\ell \\
\pi^r_{kg}\pi'_{ng}
\end{array} \right]
$$

where $g = m$ or f and $k = 3$ or 6 with $n = s$ or p respectively. The optional insertion of the triple adjoint $\mathbf{o}^{\ell\ell\ell}$ had not been considered in [BL2001]. Here are some illustrations:

$$
\text{(6.4)} \qquad
\begin{array}{ccccc}
elle & veut & se & laver & \\
\pi_3 & (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell) & (\mathbf{jo}^{\ell\ell}\mathbf{i}^\ell) & (\mathbf{io}^\ell) & \to \mathbf{s}_1 \\
\pi_{3f} & & (\pi_{3f}^r\pi_{sf}) & \pi'^r & \to 1
\end{array}
$$

$$
\text{(6.5)} \qquad
\begin{array}{cccccc}
elle & veut & se & \hat{e}tre & lav\acute{e}e & \\
\pi_3 & (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell) & (\mathbf{jo}^{\ell\ell}\mathbf{o}^{\ell\ell\ell}\mathbf{i}^\ell)(\mathbf{io}^{\ell\ell}\mathbf{p}_2^\ell) & (\mathbf{p}_2\mathbf{o}^\ell) & \to \mathbf{s}_1 \\
\pi_{3f} & & (\pi_{3f}^r\pi'_{sf}) & \pi'^r_{sf} & \to 1
\end{array}
$$

However, we cannot justify

$$
\text{(6.6)} \qquad
\begin{array}{cccccc}
elle & veut & se & {}^*avoir & lav\acute{e} & \\
\pi_3 & (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell) & (\mathbf{jo}^{\ell\ell}\mathbf{i}^\ell) & (\mathbf{ip}_2^\ell) & (\mathbf{p}_2\mathbf{o}^\ell) & \to \mathbf{s}_1 \\
\pi_{3f} & & (\pi_{3f}^r\pi'_{sf})(\pi^r\pi_{sm}) & & \pi'^r_{sm}
\end{array}
$$

nor

$$(6.7) \qquad \underset{\underbrace{\pi_{3p} \;\; (\underbrace{\pi_{3f}^r \pi_{sf}'}_{})(\underbrace{\pi^r \pi_{sm}}_{\diagup}) \;\; \pi_{3f}'^r}{}}{\text{\textit{elle veut se *avoir lavée}}}$$

since $\pi^r \neq \pi^\ell$.

7. Third person dative preverbal pronoun.

The non-reflexive *lui* and *leur* have the type

$$\begin{bmatrix} \mathbf{i}'\omega^{\ell\ell}\mathbf{i}^\ell \\ 1 \end{bmatrix}$$

the feature gender being ignored. We illustrate this as follows:

$$(7.1) \qquad \begin{array}{cccccccc} \textit{je} & \textit{peux} & \textit{la} & \textit{lui} & \textit{donner} & \\ \pi_1 & (\pi_1^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell & (\mathbf{i}'\omega^{\ell\ell}\mathbf{i}^\ell) & (\mathbf{i}\omega^\ell \mathbf{o}^\ell) & \rightarrow & \mathbf{s}_1 \\ \pi_{1m} & & (\pi^r \pi_{sf}' \pi^\ell \pi) & & \pi'^r & \rightarrow & 1 \end{array}$$

$$(7.2) \qquad \begin{array}{ccccccc} \textit{je} & \textit{peux} & \textit{la} & \textit{lui} & \textit{avoir} & \textit{donnée} & \\ \pi_1 & (\pi_1^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell) & (\mathbf{i}'\omega^{\ell\ell}\mathbf{i}^\ell) & (\mathbf{i}\mathbf{p}_2^\ell) & (\mathbf{p}_2\omega^\ell \mathbf{o}^\ell) & \rightarrow \quad \mathbf{s}_1 \end{array}$$

$$\underbrace{\pi_{1m}}_{} \quad (\underbrace{\pi^r \pi_{sf}' \underbrace{\pi^\ell \pi}_{})}_{} \;\; (\underbrace{\pi^r \pi_{sm}}_{}) \; \pi_{sf}''^r \;\; \rightarrow \;\; 1$$

The reflexive dative pronoun *se* has a more complicated type:

$$\begin{bmatrix} \mathbf{i}\omega^{\ell\ell}(\mathbf{o}^{\ell\ell\ell})\mathbf{i}'^\ell \\ \pi_k^r \pi_{sm} \end{bmatrix}$$

where $k = 3$ or 6. Here are some illustrations:

$$(7.3) \qquad \begin{array}{cccccc} \textit{elle} & \textit{peut} & \textit{se laver} & \textit{les} & \textit{mains} & \\ \pi_3 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{j}\omega^{\ell\ell}\mathbf{i}'^\ell)\,(\mathbf{i}'\omega^\ell \mathbf{o}^\ell) & \mathbf{o} & & \rightarrow \;\; \mathbf{s}_1 \\ \pi_{3f} & & (\pi_3^r \pi_{sm})\; \pi'^r & & & \rightarrow \;\; 1 \end{array}$$

$$(7.4) \qquad \begin{array}{ccccccc} \textit{elle} & \textit{peut} & \textit{se} & \textit{être} & \textit{lavé} & \textit{les mains} & (\textit{*lavées}) \\ \pi_3 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{j}\omega^{\ell\ell}\mathbf{o}^{\ell\ell\ell}\mathbf{i}'^\ell) & (\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell) & (\mathbf{p}_2\omega^\ell \mathbf{o}^\ell) & \mathbf{o} & \rightarrow \;\; \mathbf{s}_1 \\ \pi_{3f} & & (\pi_3^r \pi_{sm}) & & \pi_{sm}'^r & & \rightarrow \;\; 1 \end{array}$$

where the default subscript *sm* ensures that the past participle is *lavé* and not *lavée* or *lavées*.

$$(7.5) \qquad \begin{array}{cccccc} \textit{elle} & \textit{peut} & \textit{se} & \textit{les} & \textit{laver} & \\ \pi_3 & (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell) & (\mathbf{j}\omega^{\ell\ell}\mathbf{i}'^\ell) & (\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell) & (\mathbf{i}\mathbf{o}^\ell \omega^\ell) & \rightarrow \;\; \mathbf{s}_1 \\ \pi_{3f} & & (\pi_3^r \pi_{sm}) & (\pi'^r \pi_{pf}') & \pi'^r & \rightarrow \;\; 1 \end{array}$$

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Note that doubly transitive *laver* has two syntactic types: $\mathbf{i}w^\ell\mathbf{o}^\ell$ and $\mathbf{io}^\ell w^\ell$, since direct and indirect object complements may occur in either order. The same goes for its past participle, with $\mathbf{i}$ replaced by $\mathbf{p}_2$.

$$
(7.6) \quad
\begin{array}{ccccccc}
elle & peut & se & les & \hat{e}tre & lav\acute{e}es & \\
\pi_3 & (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell) & (\mathbf{j}\omega^{\ell\ell}\mathbf{o}^{\ell\ell\ell}\mathbf{i}'^\ell) & (\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell) & (\mathbf{io}^{\ell\ell}\mathbf{p}_2^\ell) & (\mathbf{p}_2\mathbf{o}^\ell\omega^\ell) & \rightarrow \mathbf{s}_1 \\
\pi_{3f} & & (\pi_3^r\pi'_{sm}) & (\pi'^r\pi'_{pf}) & & \pi''^r_{pf} & \rightarrow 1
\end{array}
$$

$$
(7.7) \quad
\begin{array}{cc}
\multicolumn{2}{c}{\textit{elle peut se les *avoir lavé}} \\
\pi_{3f} & (\pi_3^r\pi'_{sm})(\pi_3^r\pi'_{pf})(\pi'^r\pi_{sm})\pi''^r_{sm}
\end{array}
$$

since $\pi'_{pf} \not\rightarrow \pi$,

$$
(7.8) \quad
\begin{array}{cc}
\multicolumn{2}{c}{\textit{elle peut se les *avoir lavées}} \\
\pi_{3f} & (\pi_3^r\pi'_{sm})(\pi'^r\pi'_{pf}\pi^\ell\pi)(\pi'^r\pi_{sm})\pi''^r_{pf}
\end{array}
$$

8. Oblique preverbal pronouns in first and second person.

The preverbal pronouns *me, te, nous* and *vous* may be accusative or dative, masculine or feminine, reflexive or non-reflexive. For example, look at *me*:

$$
\begin{array}{lll}
\text{accusative} & \left[\begin{array}{c} \mathbf{jo}^{\ell\ell}\mathbf{i}^\ell \\ \pi_k^r\pi'_{1g}\pi^\ell\pi \end{array}\right] & (k \neq 1) \\[2em]
\text{non-reflexive} & & \\[1em]
\text{accusative} & \left[\begin{array}{c} \mathbf{jo}^{\ell\ell}(\mathbf{o}^{\ell\ell\ell})\mathbf{i}^\ell \\ \pi_{1g}^r\pi'_{1g} \end{array}\right] & \\[2em]
\text{reflexive} & & \\[1em]
\text{dative} & \left[\begin{array}{c} \mathbf{j}\omega^{\ell\ell}\mathbf{i}'^\ell \\ \pi_k^r\pi \end{array}\right] & (k \neq 1) \\[2em]
\text{non-reflexive} & & \\[1em]
\text{dative} & \left[\begin{array}{c} \mathbf{j}\omega^{\ell\ell}(\mathbf{o}^{\ell\ell\ell})\mathbf{i}'^\ell \\ \pi_1^r\pi'_{sm} \end{array}\right] & \\
\text{reflexive} & &
\end{array}
$$

For *te, nous* and *vous* replace 1 by 2, 4 and 5 respectively, and for *nous* and *vous* replace s by p.

It is assumed that the speaker (*je*) knows his or her own gender as well as that of the hearer (*tu*), at least this was so before the ubiquity of e-mail. The gender of *nous* and *vous* involves a calculation that we will refrain from discussing here. Here are a few examples when the speaker is male and the hearer is female:

$$
(8.1) \quad
\begin{array}{lcccc}
& \multicolumn{3}{c}{\textit{elle veut me embrasser}} & \\
& \pi_3 & (\pi_3^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{jo}^{\ell\ell}\mathbf{i}^\ell)(\mathbf{io}^\ell) & & \rightarrow \mathbf{s}_1 \\[1em]
& \pi_{3f} & (\pi_{3f}^r\pi'_{1m}\pi'\pi) & \pi'^r \rightarrow & 1
\end{array}
$$

$$(8.2) \qquad \begin{array}{c} \textit{je dois me laver} \\ \pi_1 \; (\pi_1^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{i}^\ell)(\mathbf{i}\mathbf{o}^\ell) \quad \rightarrow \quad \mathbf{s}_1 \\ \pi_{1m} \; (\pi_{1m}^r \pi_{1m}') \; \pi'^r \qquad\quad \rightarrow \quad 1 \end{array}$$

$$(8.3) \qquad \begin{array}{c} \textit{elle peut te avoir embrassée} \\ \pi_3 \; (\pi_3^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{i}^\ell)(\mathbf{i}\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{o}^\ell) \quad \rightarrow \quad \mathbf{s}_1 \\ \underbrace{\pi_{3f} \; (\pi_3^r \pi_{2f}' \pi^\ell \pi)}\; \underbrace{(\pi^r \pi_{sm})}\pi_{sf}'^r \qquad \rightarrow \quad 1 \end{array}$$

$$(8.4) \qquad \begin{array}{c} \textit{tu dois te être lavée} \\ \pi_2 \; (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{o}^{\ell\ell}\mathbf{o}^{\ell\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{o}^\ell) \quad \rightarrow \quad \mathbf{s}_1 \\ \pi_{2f} \; (\pi_{2f}^r \pi_{2f}') \; \pi_{sf}' \qquad\qquad\quad \rightarrow \quad 1 \end{array}$$

$$(8.5) \qquad \begin{array}{c} \textit{tu dois te être lavé les mains} \\ \pi_2 \; (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\omega^{\ell\ell}\mathbf{o}^{\ell\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{o}^\ell)\mathbf{o} \quad \rightarrow \quad \mathbf{s}_1 \\ \pi_{2f} \; (\pi_2^r \pi_{sm}') \quad \pi_{sm}'^r \qquad\qquad\qquad \rightarrow \quad 1 \end{array}$$

$$(8.6) \qquad \begin{array}{c} \textit{tu dois te les être lavées} \\ \pi_2 \; (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\omega^{\ell\ell}\mathbf{o}^{\ell\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^\ell)(\mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{o}^\ell \omega^\ell) \quad \rightarrow \quad \mathbf{s}_1 \\ \pi_{2f} \; (\pi_2^r \pi_{sm}') \; (\pi'^r \pi_{pf}) \; \pi_{pf}'^r \qquad\qquad\qquad \rightarrow \quad 1 \end{array}$$

Concluding remarks.

Pregroup grammars have proved to be a useful tool for investigating sentence structure in many languages; but, aside from Indo-European ones, so far only work on Arabic, Japanese and Turkish has been published, while Bambara and Chinese are still under consideration.

This approach has faced two challenges. Since pregroup grammars are context-free, as was shown by Buszkowski [2001], how can languages such as Bambara, Dutch and Swiss German, known to be non-context-free, be treated? The best known example of a formal language that is not context-free happens to be the intersection of two context-free ones, which may be described by the direct product of two free pregroups. The second challenge arises from feature agreement in French or Italian, where a speaker may have to remember at the end of a sentence what the gender of the subject was. This can be handled with a single pregroup, as was shown by Anne Preller and Violaine Prince [2008], at the cost of attaching gender subscripts to the types of many gender-neutral morphemes. An alternative approach has been proposed by Ed Stabler [2008] and Brendan Gillon, and worked with by several of their students. This approach has been followed here.

The problem of feature agreement in French has been attacked here by performing two separate computations in parallel. The familiar computation on syntactic types is accompanied by a parallel computation on feature types. One is again dealing with the direct product of two free pregroups. I have followed the earlier feature-ignoring treatment [BL2001] as closely as possible[7], with only minor revisions concerning reflexive pronouns. As a result, the features person and number are represented in both pregroups, but gender only in the new one.

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I have agonized over what feature type to assign to adjectives and past participles. I finally decided on x^r, where $x = \pi_{ng}$ or π'_{ng}, although at one time I had favoured $x^r x$, which would have allowed me to assign feature type 1 to infinitival phrases, rather than the present π'^r.

This article was intended as my first attempt to describe feature agreement in French with the help of two parallel pregroups. It ignores many questions, the answer to which requires a bit more work. For example:

Does our type assignment account for feature agreement between a past participle and a relative pronoun preceding it?

What can be said about agreement in gender and number between a noun and an attributive adjective modifying it?

Why do gender and number become irrelevant when a noun phrase occurs as the accusative complement of a transitive verb?

How can *vous* represent the second person singular and *on* the first person plural?

Endnotes

1) They do not necessarily start from the same type assignment as in [BL2001]. In particular, they avoid double adjoints in the types of preverbal oblique pronouns.

2) G.M. Kobele; T. Kusalik, W. Pedersen and J. Vander Klok.

3) In [BL2001] the inflector had been replaced by a metarule.

4) Triple adjoints have made a rare appearance in English, and will also make one here. Some authors have avoided double adjoints, thus returning to the kind of grammar pioneered by Zellig Harris [1968].

5) In [BL2001] we had used **i** bar for $\mathbf{i}'$ and **i** double bar for **j**; but this led to some problems for the typist and may have been responsible for some typos. As in Chomskyan X-bar theory, the barred items appear higher in the parsing trees which many grammarians are fond of drawing.

6) Warning: In French these adjectives are called *adjectif épithète* and *adjectif attribut* respectively!

7) I take this opportunity to make some corrections to [BL2001], using the present notation:

On page 65, line 18 f.b., after $(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^{\ell})$ insert $(\mathbf{i}\mathbf{o}^{\ell})$.

On page 67, the first two types under $^*les+vous+offrir$ should be $(\mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{i}'^{\ell})(\mathbf{j}\omega^{\ell}\mathbf{i}'^{\ell})$.

On page 68, line 2 f.b., the type of *allé* should be $\mathbf{p}'_2\lambda^{\ell}$.

On page 78, the penultimate reference should be corrected to "Theoretical Linguistics (1925), 203-234", as in the present bibliography.

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