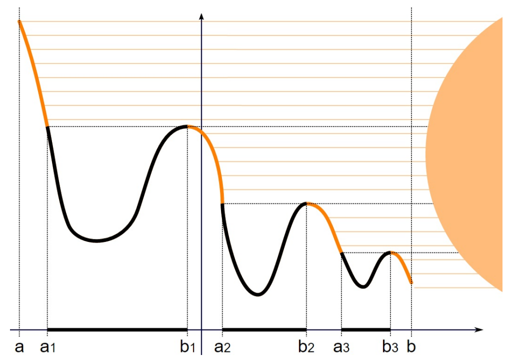
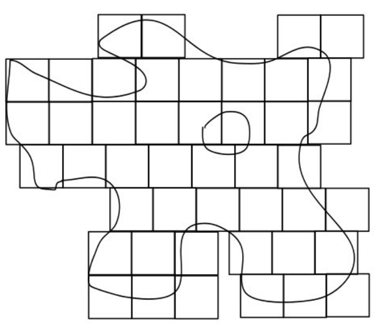
REAL ANALYSIS

Spring 2017 MATH 540 Anush Tserunyan



What is modern Real Analysis? Real analysis is the study of real-valued functions on measure and metric spaces, with emphasis on their integrability and differentiability properties. In addition to studying functions individually, it is also a study of spaces of functions, various modes of convergences, and expansions of functions in terms of more basic ones.

Why is the undergraduate real analysis not cool enough? Classical real analysis, as taught at the undergraduate level in terms of Riemann integration and continuous/differentiable functions, is completely inadequate for the modern needs of differential equations, functional analysis, probability theory, and so on. Its main shortcoming is the restrictedness of the class of functions to which it applies: for example, a pointwise limit of a sequence of continuous (or even smooth) functions $f_n : [0,1] \to [0,1]$ may already not be Riemann integrable—what a shame.



What will this course cover? It will develop modern integration theory in abstract measure spaces ($\mathbb{R}^n$ being our principal example) and modern differentiation theory for functions of bounded variation. Having built the basics, we will study L^p spaces, which provide a one-parameter family of norms for measuring the size of functions. Lastly, we will delve into Hilbert spaces and witness orthonormal expansions of functions, making the first step into Fourier analysis.

Prerequisites: Math 447 is the official prerequisite. Unofficially, students need a certain amount of mathematical maturity. If you have not studied metric spaces, you should take Math 535 before attempting Math 540.

Assessment: Weekly homework + two midterms + a final exam

Textbooks:

- R. Bass, *Real Analysis for Graduate Students*, v. 3.1 (free online)
- G. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed. (not required to purchase)